HARMONIC VIBRATIONS

AND

VIBRATION FIGURES,

BY

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PREFACE.

THIS book was originally projected in order that the experimental work of Mr. Joseph Goold, of Nottingham, on Pendulum and Plate Vibrations, might be placed on record, and the results of his life's work be made available for this and future generations. It must be admitted that at present these results are not directly utilitarian in character, although a use may be found for them some day, but the investigations have added something to the sum total of the world's knowledge, and the effects of that no man can foresee.

The manuscript was offered in turn to two of the most famous scientific publishing firms in London, and was declined by them, in both cases, on the ground that such a work could not be produced profitably, but should be undertaken by such a Society as exists in America (though, alas, not here) for the publication of works which, in the interests of Science, ought to be published, but which cannot be made to pay commercially. I, therefore, make no apology for its eventual publication by my own firm, and can only hope that the inevitable loss which my publishing friends foretell may be reduced to reasonable dimensions by a more extensive sale than they anticipate.

H.C.N.

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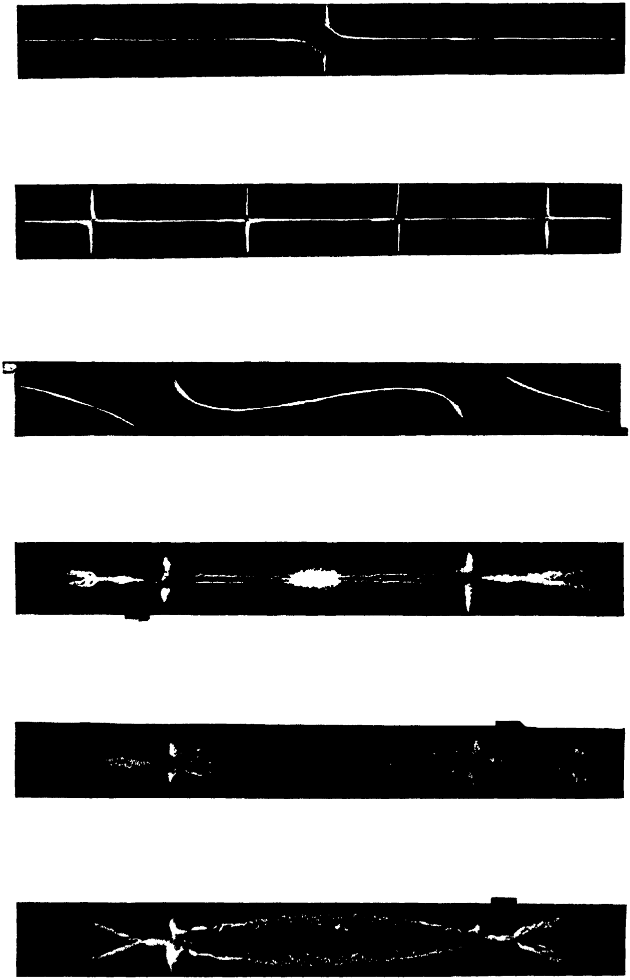


PLATE I. [*Frontispiece*

Harmonic Vibrations.

INTRODUCTORY.

BY HERBERT C. NEWTON.

SIMPLE HARMONOGRAPHS.

AS the earlier portion of this book is concerned with vibration figures produced by swinging pendulums, it will, perhaps, be best to describe first one of the simpler forms of Harmonographs, so that the action and principles common to all may be fully understood before proceeding to the more elaborate and effective machines. The expert and the scientist are fully catered for in the chapter in which Mr. Goold describes in detail the methods and results of his experiments for the benefit of those who may follow in his steps, and in the chapters by Mr. Benham on the admirable forms of pendulum instruments which he has designed; but this first chapter will be devoted to the amateur, to help him more easily to grasp the principles involved and to understand and master the instruments employed. To those who know anything of the subject, doubtless the laboured explanations of what are to them very simple facts will appear unnecessarily prolix, but if they make the matter clear to the novice, they will answer the purpose for which they are written.

Many different forms of pendulums have been arranged for the tracing of pendulum curves, the

object of them all being the same, namely, to make a permanent record in the shape of a drawing of the path taken by a freely suspended pendulum under the influence of two or more impulses.

The scientific investigator studies these for the purpose of understanding more thoroughly the laws of vibration, which can be more easily investigated with movements of such long period and large amplitude than with smaller and more rapid vibrations, for the slow swing of a pendulum is really a vibration as much as are the rapid movements of the particles of a heated body. But the amateur, apart from any scientific interest they may have for him, will find the tracing of pendulum curves the most entrancing and fascinating of hobbies; there is always the pleasure of watching to see what form the figure will take, the interest of seeing it grow and develop itself in a series of most perfect and exquisite curves, and, almost before it is finished, the wonder what the next will be like.

As one becomes more familiar with the working of the instrument, there is the pleasure of being able to repeat a figure at will, or to vary it by altering the phase, for this is an endless hobby; no one has yet exhausted all the possible combinations, the complexity and beauty of which increase as one proceeds. Some of the simpler forms are shewn in this chapter, and some of the more complicated on Plates XVI. to XXIV., but these reproductions do not in any way do justice to the delicacy and

beauty of the originals, many of the more exquisite of which cannot be reproduced at all. Apart from the pleasure he obtains himself from watching the growth and development of the drawings, there is one thing which more, perhaps, than any other induces the amateur always to be trying fresh combinations and devising new figures, and this is the attraction which his work possesses for others; his friends never seem to tire of seeing the wonderful patterns forming themselves before their eyes, and at a Conversazione there is nothing that creates so much interest, even among those people who usually consider it a bore to have to give any attention to scientific matters.

Many people whose interest in harmonic vibration was first aroused by seeing the vibration figures produced by a Harmonograph, or by a Twin-Elliptic Pendulum, have sought eagerly for some work on this subject which should be at once simple and comprehensive, to help them to go further into the matter than they have time or capacity to do unaided, and it is largely to assist such that this book has been written; partly to help them in the actual manipulation of the instruments; partly to shew what varied and beautiful figures are obtainable, and what apparatus it is best to use to produce them; and partly to make clear as far as possible the reasons why these harmonic vibrations produce symmetrical figures at all, and to explain the mathematical reasons why certain combinations

of vibrations produce certain forms, and how by previous mathematical calculation we can tell beforehand what form the figure will take. Of course, in a book written independently by two or three different authors on the same subject, there must necessarily be some overlapping, but on such a subject as the present this is probably an advantage, as any point which appears obscure as put by one writer may not improbably become perfectly clear when one reads the explanation given by another.

We have said that the object in view is to trace the path that a freely suspended pendulum would take if acted on by two or more impulses, but as a matter of fact we shall find that all the instruments in use consist of two or more pendulums instead of one, and that they are not necessarily freely suspended, that is to say, that the pendulums employed are not invariably free to swing in any direction. Indeed the simple form of Harmonograph which we shall examine first consists of two pendulums, and in our earlier experiments these will be so arranged that they can only swing at right angles to each other.

An ordinary clock pendulum is so hung that it swings in one plane or direction only, passing to and fro over the same path, backwards and forwards, and it is obvious that if we could attach a pencil to the bottom of the pendulum, and hold under it a piece of paper, the pencil would

trace a straight line on the paper, and it is of two such pendulums as this that the instrument under examination is composed. (We may ignore for the moment the fact that, as the pencil also travels in a vertical curve, or arc, of which the point of suspension is the centre, we

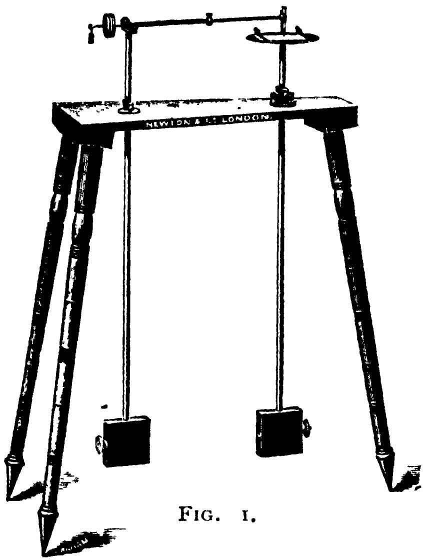


FIG. 1.

must slightly bend the paper to the same curve or else the pencil will not touch it, except for a very short distance in the centre; because in all the instruments automatic adjustments are introduced to eliminate the effects of this movement altogether.) As will be seen from the illustration (Fig. 1), the pendulums are hung from a small wooden table

supported on three legs, the pendulum rods passing through large holes in the table-top, so that they can swing without touching it, and being continued upwards for a few inches above their points of suspension. To the top of one of the pendulums a small brass table is fixed, so that it moves to and fro as the pendulum swings. The top of the other pendulum carries a long rod, at the end of which a glass pen is fitted, the length of the rod being such that when the pendulums are at rest the tip of the pen rests on the centre of the brass table. Now, as each of these pendulums can swing in one direction only, like a clock pendulum, it is obvious that, if we swing only the one carrying the pen, it will make the pen travel backwards and forwards over the paper in a straight line, passing to and fro at each swing of the pendulum, and all the result we shall get will be a straight line of ink on the paper, and not a very neat or delicate line either, owing to the pen passing so frequently along it. But it will be noticed that at each swing the pen travels a less distance than at the previous swing, that is to say, it does not go to the full length of the line, never quite reaching the ends after the first swing, but making shorter and shorter journeys along the line until it comes to rest altogether in the exact centre of the line. This is, of course, due to friction gradually stopping the pendulum. If, now, the other pendulum carrying a piece of paper on the brass table be set swinging, while that which holds

the pen remains at rest, the paper will be moved backwards and forwards underneath the pen, and again a straight line will result, but this pendulum swings only at right angles to the path of the other. and so the line on the paper will also be at right angles to the previous line, and as both start from, and come to rest at, the centre, the two straight lines will cross each other at right angles in the centre, and the resulting figure will be a cross, thus +. It is obvious that it comes to the same thing whether we move the pen or the pencil, we get our straight line just the same from either.

But it may be objected that we want to record the movement of a pendulum which can move freely if acted on by two impulses. How is the double pendulum the equivalent of this? In this way: the pen pendulum is able to move freely in one direction under an impulse in that direction, and the table pendulum is equally capable of recording an impulse given it in another direction at right angles to the first; one moves the pen and the other the paper, and we have just seen that it does not matter which is moved. So if both are moved, we get the movements of both pendulums recorded. But obviously one pen on one piece of paper can only make one line at a time. So if both pendulums are swung together at the same moment, the resulting line must be the path taken under the influence of both movements, not of either one alone. Each records all the movement

produced by one impulse, and as both unite in producing the figure, all the movements caused to the pendulums by the two impulses are represented by the figure, and this is what we wished to obtain. The object of using two separate pendulums, each to be acted on by one impulse, and then combining the two, is that it is then easier to communicate such impulses as we wish, and to vary the relative times and phases, than it would be with one pendulum alone.

Should the above description seem lacking in clearness, as the writer feels may well be the case, *perhaps it may help the reader* to refer to the remarks by Mr. Goold on this point.

It will be seen that in setting two such pendulums swinging, there are three ways in which we can vary their respective motions:

(*a*) We can impart a more violent motion to one than to the other, causing it to swing further and so trace a longer line. This is called the "amplitude" of the vibration.

(*b*) We can start both exactly at the same time; or can set one swinging and then start the other rather later, when the first has gone $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$ of its way along its path, or when it is at some certain point on its return journey. This is called the "phase."

(*c*) We can fix the weights or bobs at different heights on the pendulum rods, so that one pendulum swings faster than the other, or

both at the same height, so that they swing exactly in time or beat with each other. This is called the "rate" or "period" of the pendulum.

By keeping two of the above unchanged, and varying the third as much as possible, great changes can be made in the figures. Later on in this chapter precise directions for setting up the instrument, and hints as to the exact methods that have been found most satisfactory for manipulating and taking care of the different parts, will be given, but at present we will concern ourselves more with the results obtainable and with the connection the curves have with harmony, and their bearing on the theory of music.

Suffice it, therefore, here to say that the arm carrying the pen is hung between centres, and has an adjusting milled head at one end, by which the weight of the pen can be balanced so as to reduce the friction as much as possible, and that all the adjustments mentioned in the "Directions" must be carefully made before commencing work. Presuming all this done, let us first adjust the weights of the two pendulums to such heights that they swing exactly equally, and then start them both swinging at the same time. They are then in "unison," as it is called, and we shall get one of three figures drawn on the paper. It will either be a circle or an ellipse or a straight line, and, whichever it is, its centre will be at the point at

which the pen would rest on the paper if the pendulums were both at rest. But we have already said that if we calculate beforehand we can tell what pattern we are going to get, so why should there be any uncertainty as to what figure will result if the pendulums are in unison? The answer to this is that there is no uncertainty, all the three are really the same figure—a circle—and that the apparent difference is only one of "phase." To make this clear, take a ring (an ordinary wedding ring will do, but a larger one, such as a curtain ring, is better) and hold it up at arm's length

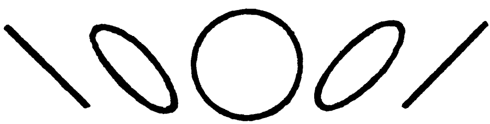


FIG. 2.

against the sky, so that you are looking straight through it, and close one eye; you see a round disc of sky surrounded by a circular ring; now tilt the top portion of the ring slightly backwards, away from you, the circular patch of sky is now of an elliptical form, and the ring itself appears elliptic, not round; tilt it still further, the ellipse becomes narrower, until, when the ring is at right angles to its first position, the ellipse of sky has quite disappeared, and the ring itself looks like a straight line. On tilting it still further an ellipse again is seen, and then again a circle.

This is exactly what occurs in our drawing. If the two pendulums are started at the same pace at the same moment when they are adjusted to unison, the figure will be a straight line, but if one commences its movement rather later than the other, we shall get the figure in a later phase, and it will be an ellipse, or, if there is a difference of exactly half the path, a circle.

But, though we talk of it as a circle, we shall find that it is not *exactly* one, as we have to take friction into account. There is very little friction at the point of suspension, but a considerable amount is created by the passage of the pen over the paper, and this, as already explained, causes each swing of the pendulum to be of less amplitude than the preceding one; hence, when the pen comes round to the point at which the circle commenced, it just misses it, and the next circle commences inside the first one, with a narrow space in between them, having the same centre but less diameter. This decrease in diameter, however, being gradual, the design is not really a succession of circles, but one long spiral commencing at the outside and ending at the centre when friction has brought the pendulums to rest. (See Plate V. for a specimen of a spiral produced by a perfect unison.) The number of turns in the spiral before it reaches the centre will, of course, depend entirely on the width of the spaces between them, and this again will depend entirely on the amount of friction, which

can be increased or diminished within certain limits by more or less accurate balancing of the weight of the pen by means of the milled counterpoise.

Another method of decreasing the effects of the friction would be by adding to the weights at the *bottom of the pendulums*, as this gives the friction more initial energy to overcome before it can bring the pendulums to rest. This additional weight would not in a "theoretical" pendulum make any difference to the rate or period of swing, which would be dependent *entirely* on the length of the pendulum, and not in any way on the weight of the "bob"; and in a "simple" pendulum, such as a leaden weight at the end of a silk thread, the effect of adding weight would be comparatively small, but in the Harmonograph which we are considering the matter is complicated by the amount of "top hamper" the pendulums carry above their point of suspension, as this has a retarding action, which must be overcome by the bottom weight; the greater the difference between the total top and bottom weights, the less will this retardation show itself in the period of swing. The weight of the rod itself also must be considered, for this will obviously modify the position of the centre of gravity in a more marked degree with a light bob than with a heavy one. There is one other matter which will affect our unison figure in practice, and that is the difficulty of getting perfect unison. The

difference between the two may be so small that, if we watch them carefully during three or four swings, they still appear to beat in unison. but after they have vibrated a dozen times the difference may be appreciable, and we may easily see that one of them starts each swing rather later than the other. The cumulative effect of this will be to gradually turn our circle into an ellipse, after which, if the difference between the two is sufficient, it will become a straight line, then again an ellipse, then a circle, once more an ellipse, and finally a straight line again, but, unless the difference is very great, the pendulums will have ceased to move long before the whole of these changes have taken place. A glance at Fig. 2 on page 10 will at once make it clear why the changes occur in this sequence. At the same time the whole figure will rotate, so that if it arrived twice at the straight line phase, the two lines would cross each other at right angles. This rotation is very evident in Fig. I. on Plate II., which is an imperfect unison figure in which the phase is gradually changing. A circle then may be looked upon as the characteristic figure of unison, that is to say, when the number of vibrations per second is the same with each pendulum.

If, while the weight of one pendulum is at its lowest, we raise the weight of the other as much as we can, we shall almost cause the latter to swing twice while the former is swinging once, but to make it

quite do so we must still further slow the beat of the long pendulum by screwing into the top of it a vertical brass rod carrying a sliding weight. The higher this weight is clamped up the rod the slower the beat will be. Clamp it fairly high up and then adjust the weight on the other pendulum till one swings just twice as fast as the other. This is represented for convenience thus 1 : 2; and unison is similarly written 1 : 1. Now on starting both pendulums we shall get a design resembling in one phase a figure 8, the various phases caused by starting one pendulum after the other being shewn in Fig. 4. It is obvious that by suitably adjusting the weights we can get various other relative rates of swing, in fact any rates of which the proportion is not greater than one to two, such as 2 : 3—that is, one pendulum making two swings while the other makes three, and in each case we shall get a characteristic design.

The two figures shewn in Plate III. are different phases of the combination 2 : 3, and it will be noted that the loops along one side of the figure are two in number, while along the subtending side there are three.

In like manner in the figures in Plate IV. which are different phases of the ratio 3 : 4 there are three loops and four loops respectively on the sides at right angles to each other. This forms a simple method of ascertaining the ratio of the vibrations by which any figure we may see was formed.

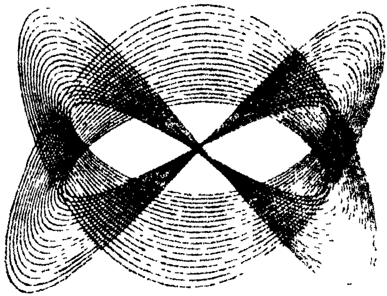


Plate III.

TWO DIFFERENT PHASES OF 2 : 3 RATIO.

See page 14.

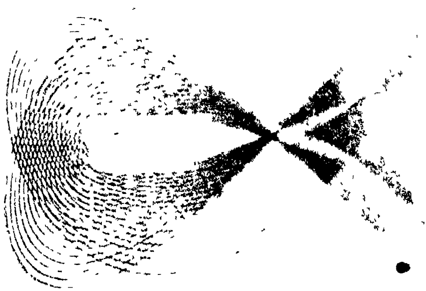
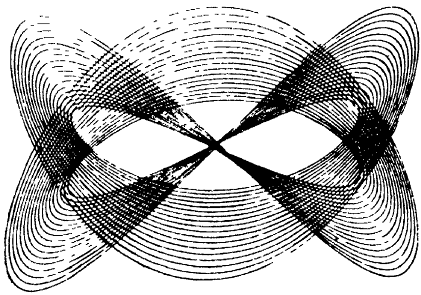


PLATE III.



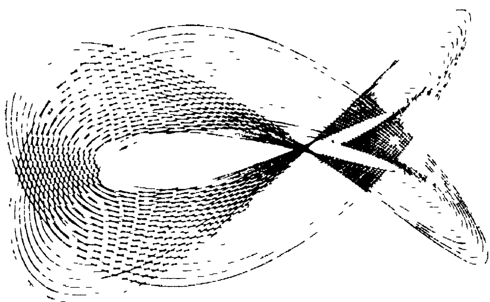


PLATE III.

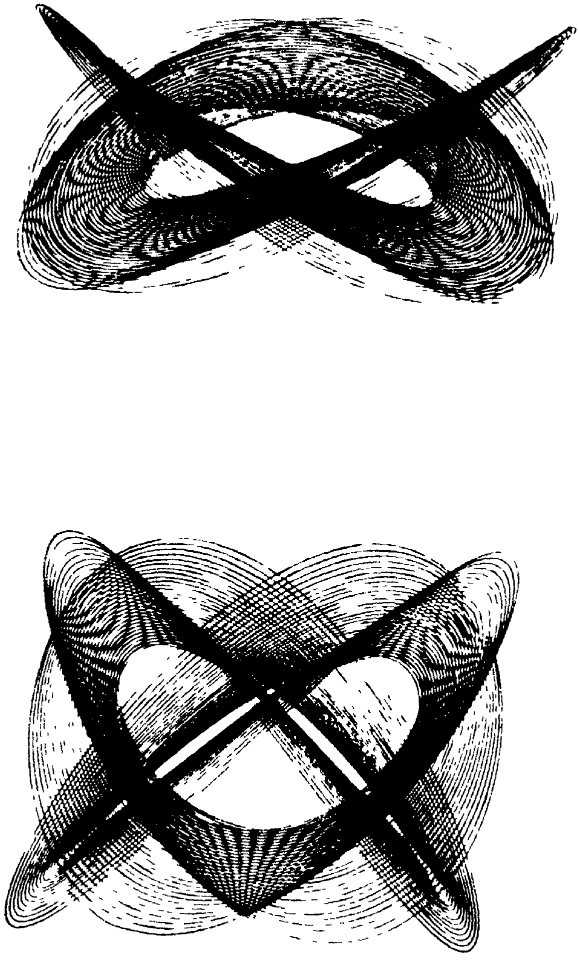


PLATE IV.

Now let us compare this with the musical scale and see how far they bear on one another.

A musical note, as is well known, is a sound caused by a series of vibrations sufficiently rapid for the ear to be unable to detect the pauses between them, and perfectly regular, that is to say, of equal duration and equally distant from each other. If two notes have each an exactly similar number of vibrations per second, we call them "in unison"; and this you will note is just what we had with the pendulums when we got our circle—each pendulum made the same number of regular beats per second.

If we have two notes, one of which has twice as many vibrations per second as the other, we call one the octave of the other, and the proportion (or the comparative number of the vibrations) is 1 : 2, which is exactly what we had in the second case with the pendulums when we got our figure 8. (The fact that this combination of curves forms an 8 instead of some other design has no connection of course with the word "octave"; the coincidence, though a curious one, is only accidental.) Between the combinations of the two fundamental notes and of the fundamental with its octave there are a number of musical intervals, some of which are given in the following table, together with the proportional numbers, or the ratios which the respective numbers of vibrations bear to each other.

Ratios		Notes.	
1 : 1	=	C : C ;	Unison
5 : 6	=	C : E♭ ;	Minor 3rd
3 : 4	=	C : F ;	Perfect 4th
2 : 3	=	C : G ;	Perfect 5th
3 : 5	=	C : A ;	Major 6th
1 : 2	=	C : c ;	Octave

If now we set our pendulums to give any of these ratios, we shall obtain the characteristic figure of that ratio; thus, if we take the ratio 2 : 3, making one of our pendulums swing twice while the other is swinging three times, we shall obtain the characteristic figure of the "perfect fifth" (see Plate III.) and we shall find that, unless we employ some of these ratios, we do not get a distinct figure at all, but that the ratios which give us harmonies in the musical scale also give us curves which combine harmoniously to form perfect figures; while if we employ discordant ratios, that is to say, ratios which are discords instead of harmonies in the musical scale, we shall as a rule also get discord between our curves, and the resulting figures will be complicated and unpleasing.

It is, I hope, now clear that our experiments with pendulums really do bear on sound vibrations, although it may perhaps be possible to carry the analogy with music too far. Mr. Benham deals with this subject more fully later on. It will suffice to point out here that if a tuning-fork gives every second a certain number of regular

vibrations gradually diminishing in length, we can investigate the laws governing such a sound by obtaining the same number of vibrations from a pendulum in a minute or in five minutes, or spread over as much time as may be necessary to enable us to make careful observations, as these vibrations are also regular and gradually diminish in amplitude; and, similarly, if we wish to investigate the combination of two musical notes, we can in like manner reproduce the number of vibrations by two pendulums.

A further series of figures can be produced by so suspending one of the pendulums that it is free to swing in *any* direction, and then communicating to it a motion compounded of two or more impulses; for instance, giving it circular motion so that the pendulum weight describes circles instead of merely swinging to and fro, while the second pendulum is swinging as before. Fig. 2 on Plate II. shews a figure drawn in this way. This opens up an entirely new field, as it is possible to vary the swing of the freely suspended pendulum in many ways. It is, of course, not so easy to reproduce any figure when one pendulum is free, but by practice more can be achieved in this direction than one would expect.

An interesting addition to this form of Harmonograph is a clockwork movement fitted to the top of one pendulum to rotate the table carrying the paper. This again gives an entirely new series of designs, some of them, which take the form more

or less of a nautilus-shell, being very pleasing. When the clock-work motion is being used, a further variation can be introduced by so adjusting the rod carrying the pen that, before commencing, it rests not on the centre of the table, but slightly to one side.

Very interesting effects can also be produced by describing two designs on the same card, one over the other. Most beautiful "watered silk" effects are obtainable in this way, perhaps the finest resulting from the superposition of two very similar or exactly similar figures. Some of these latter, drawn with the Twin-Elliptic Pendulum, are shown on Plate XXI. A curious variation can be made in these superposed figures by using a different solution in each pen in place of ink, neither of which makes any apparent mark on the paper, but which when mixed produce a coloured precipitate. The curves, therefore, show only where they cross one another, but the result is only likely to be very satisfactory in exceptional cases. A very interesting experiment requiring care and patience, but, when achieved, of permanent interest, can be performed by drawing two designs as similar as possible, but very slightly varying in phase, side by side, one in red and the other in green ink. These, looked at through a pair of glasses, one red and the other green of suitable tints, show "stereoscopically," that is to say, they stand out as if solid, and practically supply the third

dimension, which is lacking in the ordinary harmonic curve designs. If merely a 3-line figure of the principal musical intervals be prepared in this way, the series will be of real scientific value. This 2-colour stereoscopic method is more fully described by Mr. C. E. Benham later on. Another method of obtaining this stereoscopic effect is to draw the same design twice, but in slightly different phases. These designs are then placed side by side with their centres about 2½ inches apart and viewed through an ordinary stereoscope. The simplest way of obtaining the two designs with sufficiently small difference of phase, is to hold the bottoms of the pendulums back, one with each hand against weights or blocks placed on the floor, and then let them both go at once, stopping the design after it has been repeated about three times, as more simple figures produce better effects. Now, to obtain the second figure repeat the process exactly, but let go with one hand a fraction of a second before letting go with the other. Very little practice will enable the operator to obtain any difference of phase desired. It need hardly be said that none of the above-mentioned complex figures should be attempted until the earlier and simpler forms have been thoroughly mastered and can be produced at will.

Such in brief are the uses of the 2-pendulum Harmonograph, which, although incapable of producing such a bewildering variety of exquisite and

beautiful designs as are obtainable with the "Triple-Pendulum" and "Twin-Elliptic" instruments, is perhaps the most suitable for the beginner and is likely in the future to be the form most in use, owing to the same reasons that have made it popular in the past.

For example, although the Twin-Elliptic pendulum is an inexhaustible source of designs, and the 2-pendulum instrument is limited in comparison, the manipulation of the latter is so much easier,

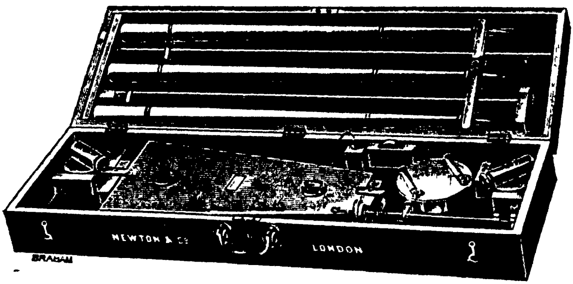


FIG. 3.

and the amount of practice requisite to achieve success is so much smaller, that the amateur would probably prefer it until he becomes "enthused," as frequently happens when one starts this most interesting and fascinating hobby, and when he has quite mastered it and desires more worlds to conquer, he can select one of the more efficient instruments.

Another reason for the 2-pendulum form usually being selected at first is that it can be used anywhere.

While the Twin-Elliptic pendulum must be hung from above and requires a fairly lofty room to produce its best effects, and owing to this and the rigidity required in the bracket or other support, cannot readily be taken to a friend's house for the evening, the 2-pendulum arrangement stands on its own legs on the floor, is only about three feet high, and packs for carrying about in a box not much more bulky than an ordinary violin case, as shown in Fig. 3.

This pattern is made in two sizes, the smaller of which, shewn in Figs. 1 and 3, gives the various figures up to, but not including, the octave, while the larger and more expensive gives the octave also. The figures on Plates II., III. and IV. were all made with the smaller pattern Harmonograph. The three-pendulum form on the other hand is more expensive than the two-pendulum instruments, and, being considerably larger, is less portable, but if expense is no object, it is a very fine piece of apparatus, and will do all that the simpler ones will, and much more.

DIRECTIONS FOR USING THE 2-PENDULUM HARMONOGRAPH.

After screwing the three legs into their places, spread them out a little and then press lightly on the table, so as to drive the points slightly into the floor and secure a rigid base, as freedom from external vibrations is essential to the production of

perfect figures. One of the brass pendulum rods has a longer top portion than the other, pass this rod downwards through the hole at the wide end of the table so that the points on the rod rest in the cups of the brass ring on the table, and clamp one of the weights on the lower end of the rod. The long arm carrying the pen is then fitted into the upper end.

On the points at the narrower end of the table the bronzed brass ring is to be fitted. This, it will be seen, has, in addition to certain cup-like depressions, 2 deep holes on one side into which the points fit. Now pass the second pendulum rod through the table, allowing the points on the rod to rest in the cups in the raised portion of the bronzed ring. To the lower end clamp a weight, and to the upper fit the brass table.

It will now be seen that each pendulum will swing in one plane only and that the planes are at right angles to each other.

Additional weights may be added to each pendulum with advantage.

PENS.—The best pen is one made of glass tube drawn out to a point, and the best method of filling it is to insert the point into the bottle of special ink and suck at the large open end. After a little has been drawn in this manner into the fine bore of the point, more may be added by means of an ordinary " filler " at the large end.

Stylographic pens may also be used, or steel

pens fitted to a light penholder, but in this case it will be found advisable to fix the pen at an angle, whereas a glass pen should always be placed vertically. A rubber band is the best and simplest means of holding the pen firmly to the socket.

Glass pens, though more satisfactory, need care. The ink should never be allowed to dry in them, and care must be taken that the ink used is free from grit, as it is difficult to clear the fine capillary tube if it once gets choked. It is best to keep the pens always in water when not in use, and should one get stopped up, it may be cleared by placing the tip in nitric acid.

INK.—As the bore of the pen necessarily varies it is advisable to have more than one sort of ink, some thicker and some thinner, as thin inks run too fast through large bores and thick inks will not pass through the finer pens. The inks should be also of different colours, both for the sake of variety and also to distinguish them.

CARDS.—The figures may be drawn upon either paper or card, but the latter is more convenient, as its stiffness keeps it flat on the table.

The surface should be quite smooth, but not highly glazed, as in its passage over the card the pen is apt to scratch up small particles of the material with which the surface is loaded, and which tend to choke the bore. In fact, until one becomes very expert, it is best to purchase such

items as cards, ink, pens, etc., from the Harmonograph makers.

UNIVERSAL MOTION.—When it is desired that one pendulum should swing freely in all directions the bronzed ring must be lifted off the pins and rotated half a turn, so that the two cup-like depressions in the recessed portion of the ring rest on the top of the points. Then the points on the pendulum should rest in the cups on the recessed parts of the top surface of the bronzed ring at right angles to the lower points. This, it will be seen, brings both sets of bearings on the same level.

BALANCING THE PEN.—When the pen is filled and ready for work, it is necessary to balance it by screwing the milled brass counterpoise weight nearer to, or further from, the pen, which should press lightly upon the paper. If it presses too lightly it will have a tendency to " bounce " or "jump," and the line will not be continuous, but pressing too heavily means increased friction, bringing the pendulum to rest sooner, and producing wider spaces between the lines, besides quickly wearing out the glass pen.

In order that the pen may be kept out of the way while the card is arranged under the springs of the brass table, a separate little weight is hung by a small cord from the hook at the end of the counterpoise screw. This is lifted and placed on the wooden table when the instrument is at work and while the counterpoise is being adjusted, but

when allowed to hang free it lifts the pen from the paper and holds it up out of the way. Care is necessary in starting the figures ; it is usually best to lower the pen gently on to the card by hand, and with a little practice this may be done without affecting the swing of the pendulum.

PART I.

DESCRIPTIVE AND PRACTICAL DETAILS AS TO HARMONOGRAPHS.

BY CHARLES E. BENHAM.

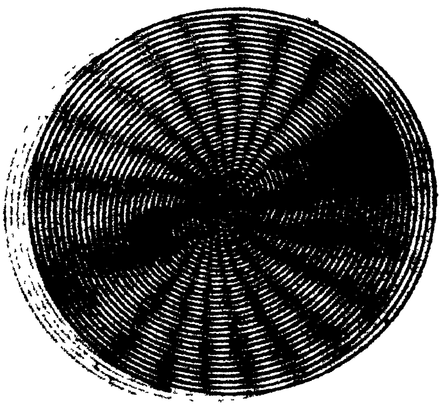


PLATE V. (B).

(I.) BRIEF HISTORY OF THE HARMONOGRAPH.

THE Harmonograph is an instrument for recording the curves resultant from the compounded movements of two pendulums oscillating at right angles. Long before it was invented, curves not unlike those produced by the pendulums had been described by means of the geometric pen invented by John Baptist Suardi, who in the eighteenth century published a work containing an enumeration of no less than 1,273 complex figures produced by his instrument.

BLACKBURN'S PENDULUM.

The earliest form of the Harmonograph was Professor Blackburn's pendulum, which was first

FIG. 4. LISSAJOUS'S FIGURES.
Successive Phases, 1 : 2

shown in 1844. This consisted of a heavy weight, hanging by a cord, and forced by its attachments to perform an orbit which compounded two rectangular paths. The curves were traced in sand or lampblack by a pointer attached to the lower side of the weight.

LISSAJOUS'S FIGURES.

In 1857 Lissajous showed similar curves resulting from the compounding of the vibrations of two tuning forks held at right angles. Mirrors affixed

to the tuning forks received and reflected a beam of light, and the resultant curves were thus projected optically on a screen.

WHEATSTONE'S KALEIDOPHONE.

Sir Charles Wheatstone produced similar effects by means of his kaleidophone, a bent steel rod with a bright bead attached, showing, when set vibrating, a curve of harmonic character by the reflection of a lamp or candle placed near the bead.

THE CLOCK-SPRING KALEIDOPHONE.

A simple way of showing curves of exactly the same character is to mount a bright bead or a round headed silver plated or nickel plated nail at the end of a piece of clock-spring about nine inches long. The spring is held in a flame near the centre, and when red-hot is twisted so that the two halves are at right angles to each other. The lower portion is gripped in a vice, and when the spring is drawn aside diagonally and released the bright bead shows by the light of a candle a curved line which, if a harmony is perfectly established, preserves its form, merely diminishing in size, until the spring comes to rest. Harmonic ratios depend upon the position of grip in the vice, which must be adjusted by trial. By the light of a coloured lamp in an otherwise dark room the effects are very beautiful. If two differently coloured lamps—say, red and green—are placed one on each side of the bead the curve will be duplicated in interlacing red and green lines.

THE STRIP RECORDER—A USEFUL DEVICE.

This simple clock spring harmonograph may actually be made to record a tracing on smoked paper. To do this, instead of the bead, or as well as the bead, let there be a cork firmly fixed at the top of the spring and on the flat upper surface of the cork fasten with glue or with two pins a strip of paper about three inches long and an inch wide. At the other end of this paper strip a short needle point must be attached vertically to the under side. The attachment is easily accomplished with a drop of hot glue allowed to run down the needle point after it has been pushed through the strip into position. The strip is thus a recording lever, very light and frictionless, and free from any tendency to bounce, and it will be found that with a little practice it can be made to trace the record of the curve perfectly on smoked paper. The strip is supported on a knitting rod, held horizontally under it while the spring is drawn aside, and let down on to the smoked paper at the moment of release. In an instant the harmonic curve appears exactly as if it had been traced with the Tisley harmonograph. The value of the paper strip lever is considerable for recording movements of any kind that necessitate absence of friction, and at the same time require that the needle shall not jump, as it would if a stiff counter-balanced lever were used. A Tisley harmonograph may be made in miniature form, with weights not

exceeding 1lb., if the records are made on smoked paper with the strip recorder.

THE DISC KALEIDOPHONE.

Another method of showing the harmonic curves optically is as follows:—Mount on a firm stand two vertical steel wires about half-an-inch apart and about twelve inches long. Firmly attached to the top of each of the steel wires is a blackened disc of card or metal, so that the discs are parallel. Each disc is about four inches in diameter, and running diagonally through the centre is a narrow slit, the respective diagonals being at right angles. When the discs are at rest there will be a central white spot at the intersection of the slits when they are viewed against the light. This spot will develop into a bright line when one of the discs vibrates, and also into a similar line, but at right angles, when the other vibrates. When the two vibrate together the line will become a compound vibration curve, and by appropriately weighting one of the wires with a movable clamped weight, adjustable at different positions along the wire, various harmonies may be produced exactly like Lissajous's figures. This simple little contrivance also illustrates very prettily the sympathy of harmonic vibrations. Directly one of the springs is started the other, if tuned in harmony, also commences to vibrate by the accumulation of infinitesimal harmonic impulses from its companion.

HUBERT AIRY'S EXPERIMENTS.

After Wheatstone's Kaleidophone no further advances of any importance were made in the subject of these compounded vibration curves until some interesting observations by Mr. Hubert Airy in 1870 led to the idea of constructing an instrument which was the actual progenitor of Tisley's Harmonograph. Mr. Airy had casually noticed that a shoot from an acacia tree when set vibrating by drawing it aside and releasing it described a very curious orbit as it came to rest. By tying a light pencil to the twig he actually succeeded in making it record its movements on paper, and these tracings were reproduced in Nature in 1870, as also were some more perfect figures which he subsequently produced by means of a crudely constructed compound vibration pendulum of long jointed rods supporting a weight of 50lb.

THE HARMONOGRAPH AND SYMPALMOGRAPH.

In 1874 Mr. S. C. Tisley and Mr. Spiller, of London, showed the harmonograph in practically its present form. This was followed by Mr. Viney's simplification called the Sympalmograph, made by Browning. Many further modifications have since been devised for obtaining special results and more complex patterns.

(II.) THE RECTILINEAR HARMONOGRAPH.

IN *Tisley's harmonograph two pendulums are so arranged that while swinging at right angles* the combined movement is exhibited and permanently recorded in the form of beautifully symmetrical curves traced on paper or card with a pen. The pendulums are so constructed that their relative lengths can be varied, with corresponding differences in the form of the curves described. All the modifications of the curves traced by the instrument are due to modifications of *period* and modifications of *phase*, two terms which must at the outset be clearly understood and distinguished by the experimenter.

DIFFERENCES OF PERIOD—THE ANALOGY OF MUSICAL CHORDS.

The differences of pattern produced with the harmonograph arise from the fact that by altering the length of one of the pendulums we alter its time-period. If the two pendulums are swinging in equal time, we may obtain by their combined influence on the pen a spiral—the unison figure. If we alter the length of one of the pendulums so that in the time that one has accomplished three oscillations the other has exactly performed two, we have a totally different figure—one with two loops, or "nodes," bounding two of its opposite sides and three of these nodes bounding the other two

opposite sides. Whatever the ratio of the two pendulums' time-periods may be, it will always be expressed by the nodes in this way. With a ratio of 3—4 we shall have 3 nodes and 4 nodes at right angles to them; the ratio 5—8 will similarly show 5 nodes and 8 nodes, and so on. If the ratio is one of very high numbers, such as 13 to 35, the figure will lack symmetry. It will have so diminished in size before it has completed so extensive a series of loops that the harmony of the design will not be apparent, and the figure will be what is called a discord, though it should be borne in mind that such figures are not really discordant, but are just as truly the product of harmonious law as the figures which, because their simple symmetry is manifest, we call harmonies. The so-called harmonies, therefore, are merely the combinations of ratios of small numbers, while the so-called discords are more or less completed combinations of ratios of higher numbers, the cycle being too protracted and involved for the eye to appreciate. Pope's line, "All discord harmony not understood," is scientifically accurate. The principle has its parallel in music, in which, as is well known, the harmonious chords are those which combine sound vibrations whose periods are in ratios of small numbers, while the combination of two notes whose ratios of vibration are represented by high numbers produces an effect of discord on the ear. The parallel must not, how-

ever, be pushed too far, for though many of the harmonograph figures correspond with musical chords, as regards the ratio of their component elements, many of them do not and some actually have no equivalents in any combination of notes on the diatonic scale.

This will be rendered clearly manifest by the following table, which gives some of the principal harmonies, with their musical chord equivalents :—

Ratio of period.	Chord in music.	Component musical notes.
Equal	Unison	C C
5 to 6	Minor third	C E♭
4 to 5	Major third	C E
3 to 4	Major fourth	C F
5 to 7	No equivalent in the diatonic scale	
2 to 3	Major fifth	C G
3 to 5	Major sixth	C A
1 to 2	Octave	C c
1 to 3	Perfect twelfth	C g
1 to 4	Double octave	C c′

It will be seen from this list that the 5 : 7 harmony, which is quite as pleasing to the eye as any of the others, has no musical equivalent chord. Besides, while in music some of the chords specified above are much more satisfying to the ear than others—for example, the major third is far more pleasing than the major fourth—it cannot be said that the curve representing the ratio 4 to 5 of the harmonograph is any more satisfying to the eye than that for the ratio 3 : 4. Therefore, as already

stated, the analogy of eye and ear must not be pushed too far, and it is rather misleading to call the harmonograph figures "music made visible," as they have been styled sometimes.

In view of the musical analogy it is always advisable to record harmonies upwards, that is to say, 2 : 3 not 3 : 2, 1 : 4 not 4 : 1, just as in recording a musical chord the fundamental note is placed first.

The time period of a pendulum with a moveable weight is most easily ascertained by experiment and not by calculation. Theoretically, in the case of a "simple" pendulum—that is, an imaginary pendulum without weight except at the bob—the period of oscillation varies inversely as the square root of the length, *i.e.*, if the length be increased fourfold, the time period will be one half what it was before. A simple pendulum does not, however, exist practically, and though the law above enunciated may serve to give a general idea of the lengths corresponding to different time periods, these are really only to be arrived at with accuracy by trial.

DIFFERENCES OF PHASE.

The figures of the harmonograph differ not only according to the ratios of period, but also according to the different phases of the oscillations. These phases are determined by the relative directions in which the pendulums are swinging. They may be —say, in the case of the unison figure—each at the

end of their swing at the same time, in which case the resultant figure is merely a diagonal line, in accordance with the well-known law of the parallelogram of forces, or on the other hand one pendulum may be at the end of its swing when the other is in the middle of its path, in which case the resultant curve is a spiral. These are the two extreme cases of phases, and may be called the *cusped* and *open* phases respectively. Between these extreme varieties of phase there are innumerable intermediate forms partaking of the nature of each. The cusped phase may be known in any of the figures by the pointed extremities which prevail in two of the nodes, while the open phases are manifested by the rounded and uniform loops. The cusped phase is the best test of the accurate harmonising of a figure. If the harmony is perfect, the pointed nature of the cusps will continue to assert itself throughout the figure, but if the figure deviates from true harmony it will open out its cusps more and more each time the pen comes round. Discord, indeed, involves a continuous change of phase, and it will be seen that if the pendulums are purposely adjusted for a slight discord, the pen will trace all the possible phases of the approximate harmony in succession.

In order to obtain control over the different phases, and to produce such as may be desired, see the instructions under the heading of "Stereoscopic Designs" (page 45).

(III.) PRACTICAL DETAILS.

THE GLASS PENS.

THE glass pen, when out of use. should be kept point downwards in a cup of clear. water, or in the ink vessel, over which there should be a cover to keep out dust. Fill the pen by drawing the ink up *through the point.* By this means no particles larger than the aperture can enter the pen to clog it. In drawing the ink into the pen first attach a longer glass tube to it by means of a small piece of *indiarubber tubing, and apply suction to this* attached tube. This will obviate any risk of drawing any ink into the mouth, or allowing any moisture from the mouth to enter the pen. The pen may, however, be filled at the top with a pipette or fountain-pen filler. if a tiny tuft of cotton-wool is first pushed into the pen as near the point as it will go. This acts as a filter. The little plug of cotton-wool is in any case advantageous, even if the pen is filled at the point. It promotes a gentle and even flow of the ink and saves the risk of dust entering at the top and clogging the pen.

A glass pen is made by heating a piece of tubing at the centre in a flame, and drawing it out so as to make two pointed tubes. These tubes are then sealed in the flame by holding the tip of the tube vertically and point upwards for a moment in the flame. The sealed point is then ground down carefully on a wet hone until the hole is just reached. The shoulders of the point are also ground away carefully on the

hone, for the width of the line will be that of the whole glass point, and not that of the hole merely, and often a pen that has worn down so as to give too thick a line may be restored by a few sharpening strokes on the hone. The three stages are shown in Fig. 5, in which A shows the pen drawn to a point and sealed, B shows it after grinding until the hole is reached, and C the final sharpened stage.

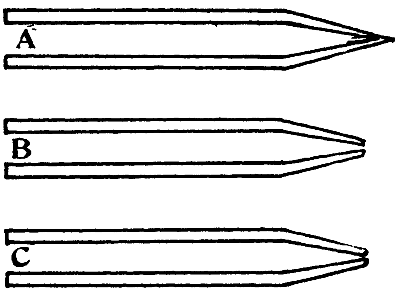


FIG. 5. GLASS PEN POINTS (MAGNIFIED)

Showing the three stages of the process of making.

The pens may be purchased from the makers of the Harmonograph, and with careful use will serve for a very long time. The pen should rest on the paper as lightly as is practicable without bouncing, the counter-balance being adjusted accordingly. A new pen often works a little harshly at first, but it quickly gets into good condition after it has been used a little while. Should the ink have been allowed to dry in the pen point, it will be quickly

cleared by placing it in dilute nitric acid, afterwards cleaning it well with water before refilling with ink.

INKS.

Morell's violet ink is one of the best to use, but most of the ordinary coloured commercial inks answer very well. A very little gum arabic added to some inks is an improvement, as it tends to prevent the lines from running together. If black tracings are required, ordinary blue-black ink may be used, or even liquid Indian ink, preferably the *non*-fixed kind, as the fixed kinds are more apt to clog the pen. In using the Indian ink there must be no trace of water in the pen when it is filled, unless it be distilled water.

Special inks, however, can be obtained of various colours which dry quickly, an important point when the lines are close together or cross each other frequently, as is the case in some of the most beautiful designs.

PAPER AND CARDS.

"Ivory" cards and Bristol board afford a good surface, as also does a good quality *smooth cream-wove writing paper*. Rough paper should be avoided, and also paper with enamelled surface, which, though it takes fine lines that dry promptly, has the disadvantage of ultimately clogging the pen with the substances with which the surface is loaded.

Very fine designs may be traced on paper coated

with a strong solution of iron sulphate and allowed to dry. Instead of ink the pen is filled with strong pyrogallol, to which a little sulphite of soda has been added. The result is that wherever the pen touches the paper an intensely black but very fine line ensues. This ink, after a day or two, becomes quite insoluble, but unfortunately it does not retain its blackness permanently.

Tannic acid in place of pyrogallol gives a permanent black, but not one of such intensity. *The advantage of using a soluble salt in place of* ink is considerable for the production of the very finest lines, and if it is for purposes of photo-engraving, the non-permanency is often of no importance so long as a good black is obtained for reproduction.

A strong solution of sugar, to which a little sulphuric acid has been added (preferably coloured with indigo), gives lines which turn to an intense black when the paper is strongly heated afterwards. The heat should be applied as soon as possible after the tracing is drawn, or the lines will degenerate to a brown in course of time.

TRACINGS ON SMOKED PAPER.

For these a needle-point vertically mounted in a wooden-holder must be substituted for the pen, not forgetting to re-adjust the counterbalance for the altered weight of the pen lever. The paper should be that with an enamelled surface, commonly used

for "process" illustrations, and popularly called "art paper." It should be enclosed in a small frame—such as a child's transparent drawing slate—and backed with blotting paper. When thus held flat it may be smoked in the flame of a wax taper held *horizontally* underneath it and moved to and fro so as not to char the paper. The curves will be in white lines on black, and of great delicacy, there being no running together of the lines at intersecting places, as with ink. To fix the tracings, *take them carefully out of the frame, and brush the back* of the smoked paper with a fixative made of hard white spirit varnish one part and methylated spirit six parts. When quite dry the surface can be varnished with a turpentine varnish or lightly-polished with a tuft of cotton-wool.

TRACINGS ON SMOKED PHOTOGRAPHIC PAPER.

Collodion-coated sensitized paper, such as Paget's self-toning paper, takes the smoke very well. Instead of fixing the tracing lay it in the light until the white lines have turned a deep brown. Then sponge off the smoke in a saucer of water and fix the print in the usual way. By this means dark lines on a white ground are produced, while any running together of the finest lines is entirely obviated.

TRACINGS ON SMOKED GLASS.

Flood a clean piece of glass with benzoline to which a little petroleum has been added,

and drain the liquid off at a corner of the plate, taking care not to be near a flame, as the vapour is inflammable. Smoke the benzolined surface in a candle flame, moving the glass about all the time to prevent fracture from the heat. The smoke first deposits as a shiny film, and presently assumes a dry velvety opaque appearance. The under stratum is still tacky and the film presents a medium admirably adapted to take the cut of the needle and give perfect lines. The tracings can be mounted as lantern slides or transparencies, thin card slips being placed under the edges of the cover glass to prevent actual contact. If it is desired to use the tracings as negatives for printing from photographically, they may be fixed be flooding the smoked side carefully with a negative varnish, and draining off at the corner. It should be borne in mind in producing these tracings on glass that the weight of the glass will probably have slightly altered the period of the pendulum on which it rests, the weight of whichmust be readjusted accordingly.

The photographer's so-called "opal-glass" takes very good lines, if smoked in a candle flame very lightly and without benzoline. The figures may be fixed with varnish, or protected by a glass covering. The film side of an unused dry plate also makes a good surface for smoking, and takes the varnish well.

CLEARING SMOKED TRACINGS OF LOOSE PARTICLES.

The needle is apt to scatter loose particles of carbon over the surface of the paper or glass. These are generally removable by a light tap on the back of the paper, but on the glass, being impregnated with benzoline, they often adhere, and are not so easily got rid of it. They should, however, be removed before fixing, and this may be easily done by taking any excited electric, such as an ebonite strip or rod rubbed with a warm flannel and holding it near, but, of course, not letting it touch the glass surface. All loose particles are attracted by the ebonite, and the lines are left perfectly clear.

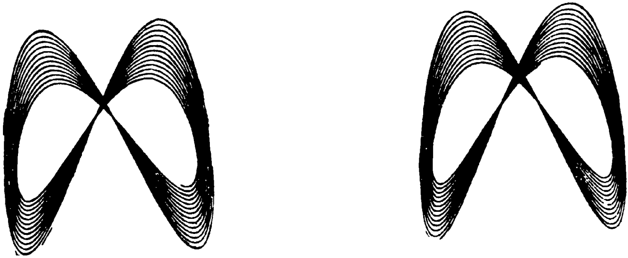
STEREOSCOPIC DESIGNS.

Stereoscopic results of a very interesting character may be obtained by tracing the same figure twice—exactly of the same size, but in very slightly differing phase. This is by no means so difficult as it might seem. Arrange the pendulums for a given ratio, say 2 : 3. Draw the lower ends of the pendulums up to firm stays on the floor, placed at such a distance as to give the required amplitude of oscillation, and having pressed them against the stays, one with each hand, let go of both at the same instant. Drop the pen, say, at the third swing of the longer pendulum. Count, say, six completed orbits of the figure on the paper and then lift the pen. If this has been done accurately the figure

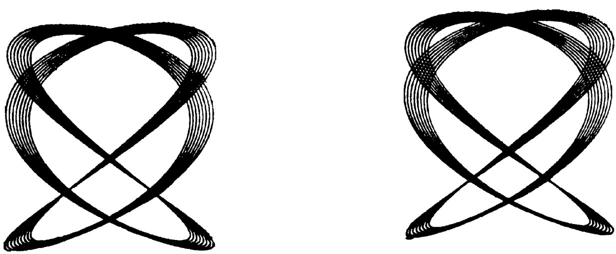
should be in its cusped phase (with pointed extremities). If the process were repeated exactly on a second card the figure would be reproduced exactly, line for line. But to obtain the stereoscopic effect it is necessary that the second figure should be slightly varied in phase; and to ensure this the two pendulums must not be released simultaneously as before, but one must be allowed to start the least fraction of a moment before the other. If this is done deftly—and it only requires a little practice—the two figures will differ in phase just *sufficiently* to give a surprising effect of solidity when the two are viewed side by side in the stereoscope. The best stereoscopic effects are, however, with the open phases. To produce these the pendulums must not be released simultaneously in the case of either figure, but with a little practice it will be found easy to allow for the requisite difference in the time of release to produce the necessary difference of phase.

In mounting for the stereoscope the two patterns should be exactly in a horizontal line, and the corresponding parts of the two figures should be at a distance apart of from 2½ to 2¾ inches.

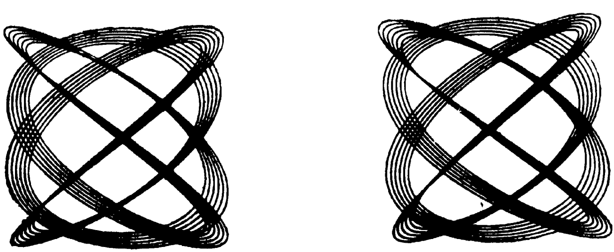
Traced with a needle on smoked glass the stereoscopic effects are astonishing, and those who see them for the first time in a good closed stereoscope will hardly believe that they are not looking at a solid cage-work of silver wires, so perfectly do the lines seem to stand out in space.



1 : 2. Stereoscopic

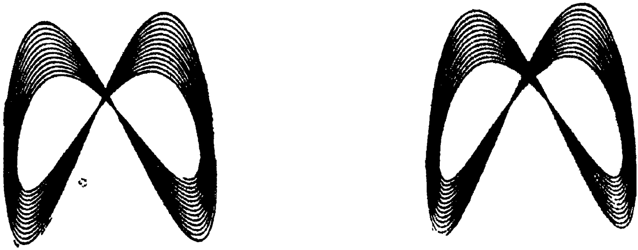


2 : 3 Stereoscopic



3 : 4. Stereoscopic.

PLATE VI.



1 : 2. Stereoscopic

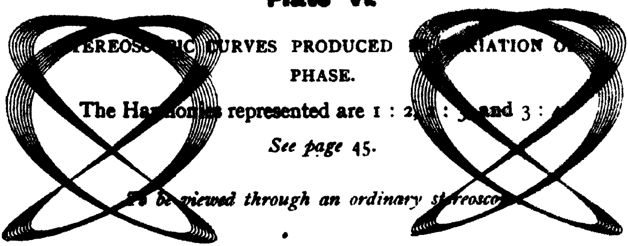
Plate VI.

STEREOSCOPIC CURVES PRODUCED BY VARIATION OF PHASE.

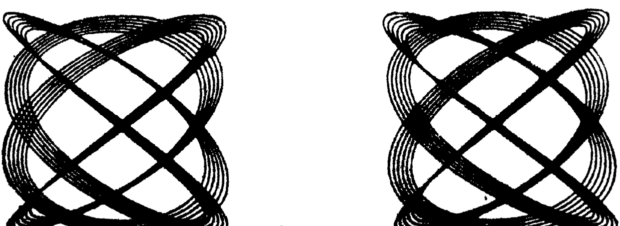
The Harmonies represented are 1 : 2, 2 : 3 and 3 : 4.

See page 45.

To be viewed through an ordinary stereoscope.



2 : 3. Stereoscopic



3 : 4. Stereoscopic.

PLATE VI.

The ratio 3 : 4 gives an effective design for stereoscopic representation, and if the harmony is intentionally made not quite perfect a still more beautiful curvature of the lines is brought out.

The stereoscopic figures should not be continued to the centre of the figure. About twelve completed orbits are the maximum that will show perfect effects, and three or four are generally preferable.

A curious difference of effect is to be observed in comparing the Figures on Plate VI. and Plate VII. In Plate VI. the pairs are arranged so that the smaller number of loops are vertical and the larger number horizontal. The resulting solid figure in these cases has lines which pass through its interior. In Figure VII., where the smaller number of loops are placed horizontally, the stereo effect is of a ring form with no lines passing toward the interior. The two effects may be shown with any pair according to the way they are mounted, and if mounted on revolving discs and turned simultaneously from vertical to horizontal, the change of form may be seen to take place as the revolution is accomplished in the stereoscope.

Another way of showing the patterns in stereoscopic relief is to draw the two phases on the same paper about an eighth of an inch apart (taking care that they are separated *horizontally*, not vertically), one in red ink and the other in pale green ink. The design is then viewed with the red and green stereoscopic spectacles, sold by Newton and Co.

ZOETROPIC DESIGNS.

With a little care, and after a little practice, it will be found possible by appropriate timing of the release of the pendulums from the stays to obtain any given phase of a figure, and a series of 16 or more may be made to represent the various modifications. If these 16 or more are placed in order in a zoetrope with a corresponding number of openings, the change from phase to phase may be actually manifested as a continuous process of movement on the part of the figure, which seems to dilate and contract, and even to revolve, as it sways from phase to phase in a strange mysterious rhythm of undulation. The best form of zoetrope for demonstrating the effect is the old-fashioned phenakistoscope—a revolving disc with a number of slits around the circumference. The series of phases, *traced boldly with a fairly broad* pen, is mounted in order, one figure between each of the slits on the disc, and as the disc is revolved the figures are viewed in a mirror through the slits.

CROSSED UNISON SPIRALS AND "WATERED" EFFECTS.

A great variety of beautiful "watered" effects may be produced by tracing a unison spiral and then repeating the figure on the top of the first with a very slight variation of phase. The intersections of the lines bring out a symmetrical pattern often of great intricacy and varying with the variation of

Plate VII.

STEREOSCOPIC FIGURES.

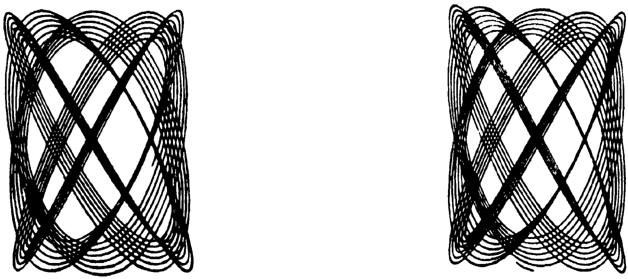
Harmonies 4 : 5 and 5 : 6.

See page 45.

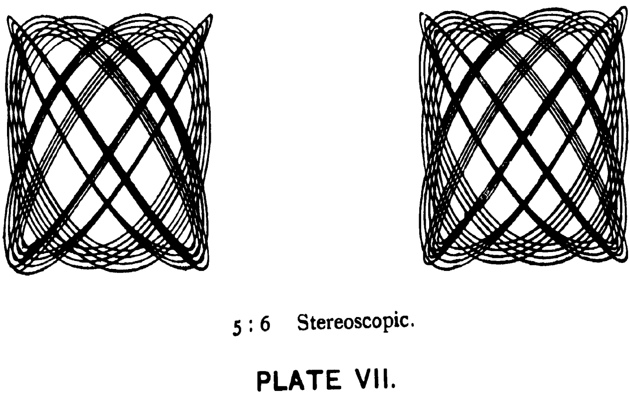
To be viewed through a stereoscope.

5 : 6. Stereoscopic

PLATE VII.

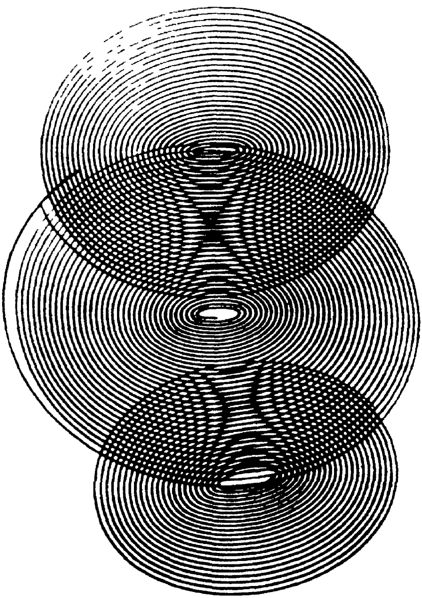


4 5 Stereoscopic.



5 : 6 Stereoscopic.

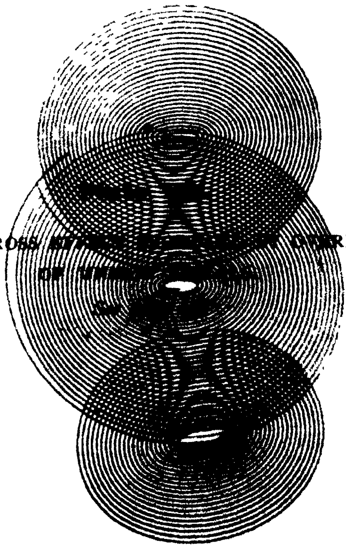
PLATE VII.



Watered Cross Effect

PLATE VIII.

WATERED CROSS [illegible] OVERLAPPING



Watered Cross Effect

PLATE VIII.

phase and the form of the spiral—whether elliptical or spiral. The watering effect is well shown by means of two superposed tracings on transparent paper. See Plate V.

THE WATERED "CROSS" EFFECT.

If the two circular or elliptical spirals are superposed, not centrally, but in this way :

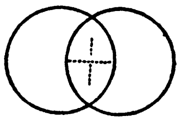


FIG 6.

the watering will take the form of a very distinct cross, which may appear white or black according to the way in which the lines intersect. This cross is very suggestive of the cross shown in certain polarisation experiments, and it is said that there is some connection theoretically between the two phenomena, which are both interference effects. See Plate VIII.

"NEGATIVE" WATERED EFFECTS.

By superimposing two unison figures that have been traced with a needle point on smoked glass, of course the effect is similar to the watering on paper, except that the lines are white on black. But if the spirals are traced on separate glasses, it is obvious that the superimposing of these will give quite another effect. In this case nearly the whole of both spirals is obliterated and the light only shows through where the actual intersections of the lines occur. In other words, the result is that the watering is shown, while the actual spirals, as such, are

not visible. This may be called "negative" watering, to distinguish it from the ordinary form. Very beautiful effects may be obtained in this way, and if the two smoked glasses are mounted as lantern projections in one of the holders that allow for the rotation of one glass, a wonderful appearance is thrown on the screen, the watering completely changing form as the rotation proceeds.

NEGATIVE WATERED EFFECTS ON PAPER.

The "negative" watering effect may be produced in colour on white paper by a very simple expedient. Trace the unison spiral in either of the two solutions used in the familiar "blue print" process of photography. When this is dry clear out the pen, cleaning it well with water, and then fill it with the other solution and repeat the spiral in this over the other. Lay the paper in a bright light, sunshine if possible, until the pattern is fully printed out. It will appear wherever the lines intersect. When the printing is completed wash well in water and the negative result will be permanently recorded in blue on the paper. Other chemical solutions might be substituted to produce similar effects, but none are much simpler to deal with than the two used for the cyanotype. They are as follows :—

(1)	Ammonio-citrate of iron ...	4 oz.
	Water	14 oz.
2)	Potassium ferricyanide ...	$2\frac{1}{2}$ oz.
	Water	15 oz.

(IV.) BENHAM'S TRIPLE PENDULUM.

BENHAM'S triple pendulum (see Fig. 7) involves three pendulums, but practically combines four rectilinear movements. Besides the two pendulums which are connected with the pen, the gimbals supporting the card on which the designs are traced give a second pair of rectangular oscillations. Thus a unison spiral is produced, alike whether the pen is alone in movement and the paper at rest or the paper in movement and the pen at rest. When both are in movement we combine the two spirals, with results of great beauty, complexity, and diversity. These two unisons thus combined may have relative differences of period and phase, and a further new feature—*difference in direction of rotation.* The two may move either *concurrently* or *antagonistically.*

By these possibilities of modification an immense field of variation is introduced into the designs.

TO USE THE INSTRUMENT AS AN ORDINARY HARMONOGRAPH.

By removing the top weights on the two long pendulums and fixing the third pendulum stationary, the instrument is the same as Tisley's, and may be used to produce the same figures by simply altering the length of one pendulum to produce the required harmonies. With this the stereoscopic and other patterns already described in Chapter III. may be very conveniently traced.

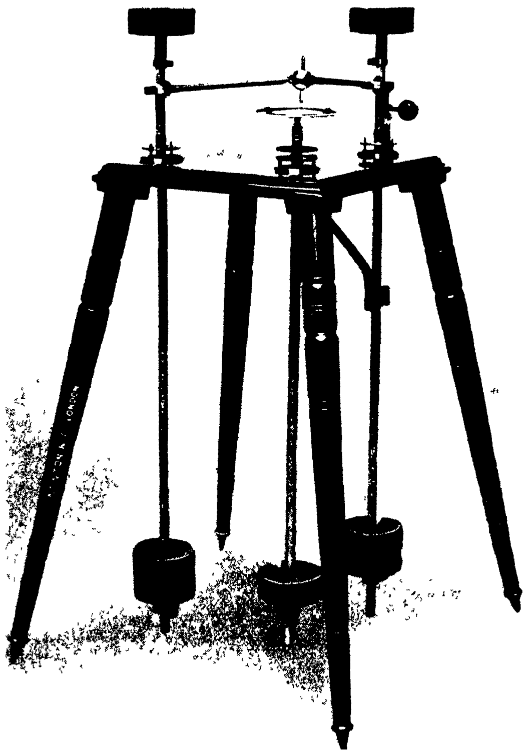


FIG. 7. BENHAM'S TRIPLE PENDULUM,

Combining four rectilinear movements by means of two rectangular pendulums and a stand mounted on gimbals.

By removing only one of the top weights combinations of much wider ratios may be produced. The beautiful 1 : 2 figure, with its wing-like cusps, and also 1 : 3, 2 : 5, 4 : 7, and other harmonies outside the range of Tisley's instrument are all available, without bringing into play the gimbal-mounted pendulum. The lengths requisite for these harmonies will soon be found by the experimenter, if he will take the trouble to time the pendulums and make a record of the periods associated with the placing of the weight at different intervals along the rod.

THE DOUBLE ELLIPTIC MOVEMENT.

To bring the third pendulum into action it is necessary that the top weights should be in position on the other pendulums, and their periodicity adjusted *until a perfect unison spiral is obtained.*

The gimbal pendulum is then adjusted in length until its period is in some given ratio with the other. 1 : 3 is one of the best harmonies, giving the greatest variety of beautiful designs.

CURIOUS LAW OF THE ELLIPTIC HARMONIES.

With regard to all the figures obtained with the three pendulums the two following interesting laws as to their character are to be noted :—

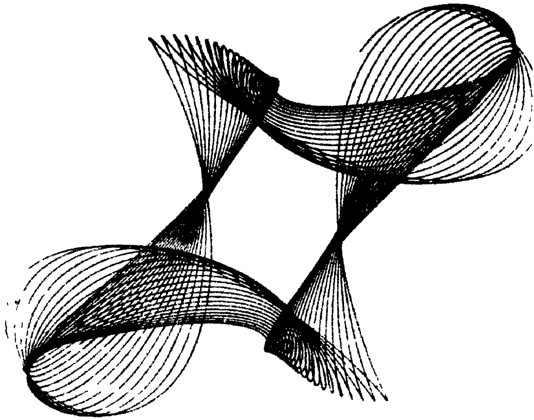
(1.) When the respective rotations of pen and paper are *antagonistic* the nodes will point *outwards* from the centre and will be equal in number to the

sum of the numbers composing the ratio. Thus a ratio of 1 : 3 will give 4 such nodes, a ratio of 7 : 11 will give 18, and so on.

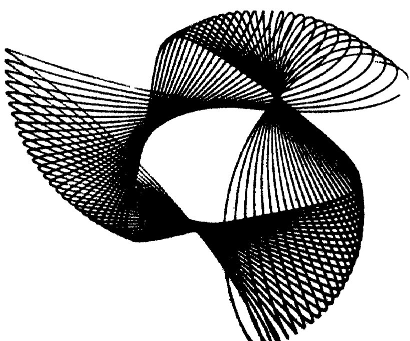
(2.) When the two rotations are *concurrent*, the nodes will be directed *inward* towards the centre of the figure, and will be equal in number to the *difference* of the numbers composing the ratio. Thus 1 : 3 concurrent will give two such nodes, 7 : 11 will give four, and so on.

From these laws the ratio in any given harmony may easily be identified. The figure has only to be traced in concurrent and in antagonistic motion, and the ratio is found. Thus, if we find the number of nodes in a given figure (antagonistic motion) to be seven, we do not know whether it is 1 : 6, 2 : 5 or 3 : 4, but by tracing it concurrently the problem is at once solved, for in the first case we shall find the internal nodes amounting to 5, in the second case to 3, and in the third case to 1. Indeed, in the case of such harmonies as 4 : 11 it is only by tracing the figure concurrently and antagonistically that we can arrive at the ratio, which is then easily computed by this simple rule:—Add together the numbers of the concurrent and antagonistic nodes. Half this sum gives one of the figures of the ratio.

Ex.:—Given a figure that displays 19 nodes antagonistically and 3 concurrently. What is the ratio? $19 + 3 = 22 \therefore 11 =$ one number of the ratio, and consequently 8 is the other.



3 Symmetrical Harmony



1 : 2. Unsymmetrical Harmony.

PLATE IX

Plate IX.

EXAMPLES OF SYMMETRICAL HARMONY (1 : 3) AND UNSYMMETRICAL (1 : 2).

See page 55.

The rationale of this simple rule is apparent on considering the problem as a simple algebraic equation:—

$$\begin{aligned} x + y &= 19 \\ x - y &= 3 \\ \therefore 2x &= 22 \\ x &= 11 \end{aligned}$$

SYMMETRICAL AND UNSYMMETRICAL HARMONIES.

Another remarkable feature of the harmonies of compounded elliptical movement is that they are of two sorts, those that are *symmetrical* and those that are *unsymmetrical* and yet still harmonious.

It will be found that whenever the sum or difference of the ratio numbers is *even* the harmony is symmetrical, and when it is *odd* the harmony is unsymmetrical. The symmetrical forms are, of course, much more beautiful than the others, and it will be seen that they are much fewer in number, for as the ratios have to be those of small numbers, all that are practically available are 1 : 3, 3 : 5, and 3 : 7, with perhaps 5 : 7, whereas 1 : 4, 1 : 2, 4 : 5, 5 : 6, 6 : 7, 2 : 5, 2 : 7, and 4 : 7, are all factors of unsymmetrical harmonies. The varieties of phase are, nevertheless, so innumerable that even one harmony, such as 1 : 3, the most effective of all, yields an almost infinite variety of figures. It may be asked why there should be so many more ratios totalling to an uneven number. The reason, as will be seen on reflection, is that all ratios in which

both numbers are divisible by 2 must obviously be excluded. If these could be admitted as separate ratios, there would be as many even as odd totals.

HOW TO UTILISE UNSYMMETRICAL AND HIGH RATIO HARMONIES.

Unsymmetrical harmonies, however, and those of high numbers may be described with beautiful effect—especially in antagonistic rotation—if the amplitude of the orbit of the gimbal-mounted pendulum is kept within very small compass. The phase of this, and more particularly the orbit of the pen-bearing pendulums, should be such as to describe a spiral as nearly as possible circular. The slight modifications of the third pendulum will then produce an exquisitely interlaced figure with radiating lines and a beautifully watered pattern. The harmony 4 : 11, which is a little away from the ratio 1 : 3, is a specially good one for these designs.

SIMPLE METHOD OF FINDING THE RATIOS.

To save time in adjusting the instrument for any required ratio, it is advisable to fill in the following table, testing each pendulum by counting its oscillations for a minute at the several positions of the weight specified. In doing this it is well to have the pen working, so that the conditions are just the same as in actual practice. It may seem a long process, but, once accomplished, it is so easy

to find any desired ratio at once, that the preliminary trouble is well worth while.

TABLE OF PENDULUM PERIODS.

Pen-bearing pendulum, full length, with top weights

No. of oscillations per minute ... ...

Ditto, without top weight—

No. of oscillations per minute .. ...

Ditto at inches per minute ... ...

Ditto at ditto ditto ... ...

Ditto at ditto ditto ... ...

Ditto at ditto ditto ... ...

Ditto at ditto ditto ... ...

Ditto at ditto ditto ... ...

Ditto at ditto ditto ... ...

Ditto at ditto ditto ... .

Ditto at ditto ditto ... ...

Ditto at ditto ditto ... ...

Ditto at ditto ditto ... ...

Ditto at ditto ditto ... ...

Ditto at ditto ditto ... ...

Ditto at ditto ditto ... ...

Ditto at ditto ditto ... ...

Ditto at ditto ditto ... ...

Ditto at ditto ditto ... ...

Ditto at ditto ditto ... ...

Ditto at ditto ditto ... ...

Ditto at ditto ditto ... ...

GIMBAL MOUNTED PENDULUM.

Weight at inches.	No. of oscillations per minute
Ditto	ditto .. ..
Ditto	ditto... ... ...
Ditto	ditto... ... ...
Ditto	ditto... ... ...
Ditto	ditto... ... ...
Ditto	ditto... ... ...
Ditto	ditto... ... ...
Ditto	ditto... ... ...
Ditto	ditto... ... ...
Ditto	ditto... ... ...
Ditto	ditto... ... ...
Ditto	ditto... ... ...
Ditto	ditto .. ..
Ditto	ditto... ... ...
Ditto	ditto .. ..
Ditto	ditto... ... ...
Ditto	ditto... ... ...
Ditto	ditto... ... ...
Ditto	ditto... ... ...
Ditto	ditto .. ..

It is evident that with these data any required harmony may be at once approximately found. To get the pendulums more exactly in tune trial should be made with the weights at the approximate distance, and if it is found that the repeat line *lags behind* at the nodal points, the weight of the gimbal mounted pendulum requires *lowering*. If the repeat

line falls *in advance* at each round the weight requires *raising*.

For the more complex harmonies—such as 4 : 15—the greatest exactness of period is essential, and the securing of the correct length may require several trials, but for the simpler ratios—such as 1 : 3—such exact harmony is not even desirable. Most of the simpler curves are more beautiful with a very slight element of discord introduced, though, of course, the discordance must not be sufficient to actually interfere with the symmetrical effect.

THE CHARM OF SLIGHT DISCORDANCE.

To illustrate the beauty of such slight discordance, a resource is provided in the Benham triple pendulum which opens up a fresh field of experiment of a very interesting character. It will be noticed that the gimbal ring has cup-shaped depressions for the pendulums, in order that the two point bearings may be at one and the same level, and a perfect unison be ensured. If, however, the ring is turned round so that the bearings rest not in the cups, but on the flat parts of the ring, a slight divergence from unison will be introduced, and by doing this in the case, say, of a 1 : 3 pattern, and also throwing the other two pendulums a very little out of unison by adding a very light extra top weight—say, an ounce or half an ounce—a marvellous diversity of new forms is obtainable, some of which are extremely curious.

ADAPTING THE AMPLITUDES.

The skill of the worker is chiefly exercised in ensuring that the *two sets of pendulums shall finally come to rest at the same time.* If one set comes to rest before the other, the central portion of the figure relapses into a mere spiral, and it should be the aim of the worker to allow such amplitude of oscillation as will obviate such results. By skill in this matter the expert is distinguished from the tyro in the eyes of the initiated. It is clear that the appropriate amplitudes may be discovered by testing the pendulums at differently extended orbits until it is found at what relative amplitude they come to rest together.

THE UNISON FIGURE.

With the top weights it is obviously impossible to get the two spiral movements of pen and paper in unison, for the gimbal mounted pendulum is too rapid in its oscillations. Yet it is particularly desirable to secure this figure, as it belongs to the symmetrical category. To obtain it, the top weights must be removed, and the lower weights re-adjusted until all three pendulums are in complete accord. A fine shell-like figure is then the result with opposed rotation. By adjusting the pendulums so that they are very nearly at unison, but not absolutely so, a variation of the phase takes place as the figure proceeds, and some fine curves may be obtained.

TRIPLE RATIOS.

It is also practicable to obtain some curious figures by arranging the long pendulums at such a ratio as 1 : 2 (removing one of the top weights only), while the pendulum on gimbals is at a ratio of 1 : 3 with the slower of the other two, and 2 : 3 with the other. These complex figures, however, though interesting and curious, are few in number, and are not particularly graceful patterns.

STEREOSCOPIC WORK WITH THREE PENDULUMS.

Stereoscopic figures may be produced with the Benham Triple Pendulum by the same method as that already described, but it requires some adroitness to liberate the three pendulums so as to ensure exactly the requisite difference of phase, and it should be noted that in doing so it is obvious that the path of the gimbal pendulum has to be confined to a straight line instead of a rotating orbit.

(V.) BENHAM'S MINIATURE TWIN-ELLIPTIC PENDULUM.

THIS is one of the simplest forms of a twin-elliptic pendulum. It consists of a gimbal-mounted pendulum with a flat top like the one in Benham's Triple Pendulum (see page 51), except that it terminates in a ferrule centrally pierced, with strands of silk passing through the hole. This ferrule is exactly provided in the screw

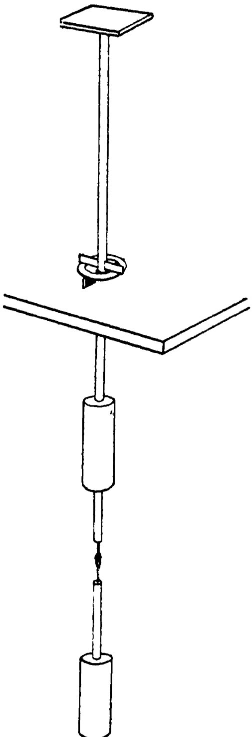
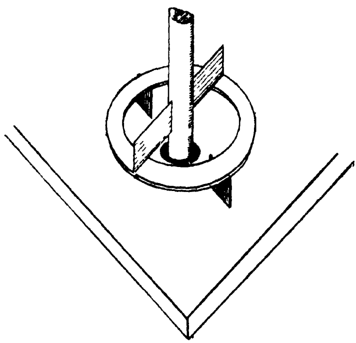


FIG. 8.

BENHAM'S MINIATURE TWIN-ELLIPTIC PENDULUM.



ENLARGED VIEW OF GIMBAL MOUNT SHOWING RING AND KNIFE EDGE.

blind cord-holders sold at any ironmonger's, and which consist of a conical brass piece

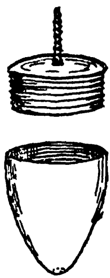


FIG. 9.

centrally pierced and a cylindrically fitting screw to attach the cone to the pendulum (Fig. 9). By a hook at the other end of the silk strands a second weight is hung. This weight is mounted on a short rod with a clamp to allow for its being raised and lowered. The tracing is done on the top of the first pendulum with a lever pen, of form as shown in Fig. 10. Adjustments are made by raising or lowering either of the weights, by adding a top weight or by lessening the amount of the respective weights. The figures obtained are similar in character to those obtained

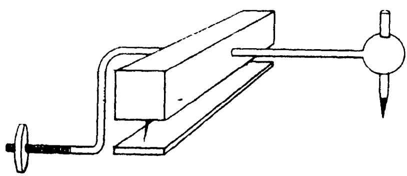


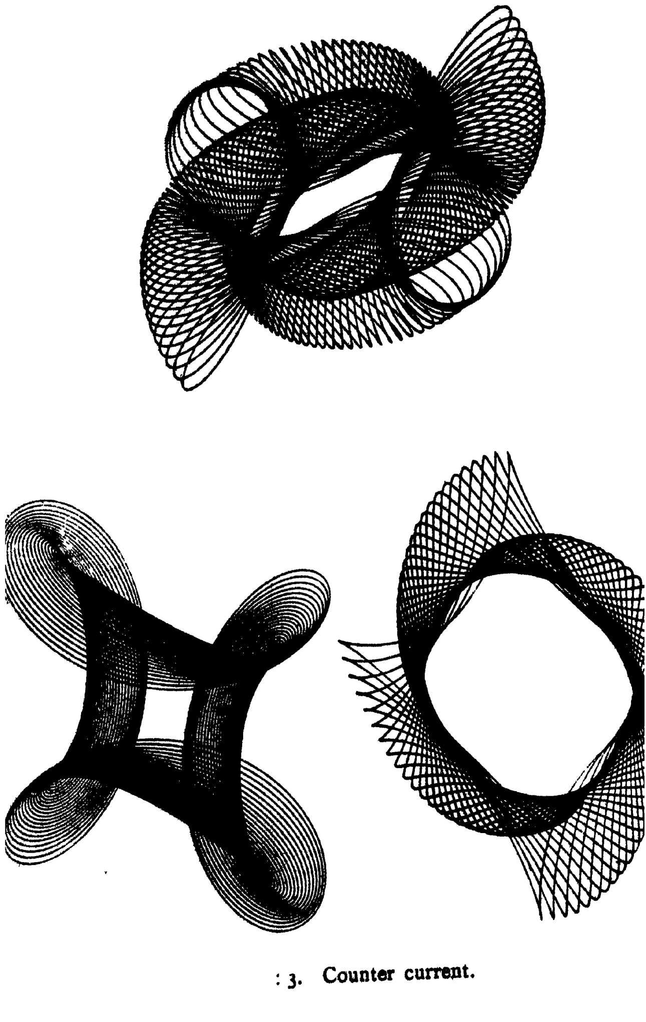
FIG. 10. PEN LEVER FOR BENHAM'S TWIN-ELLIPTIC PENDULUM,

consisting of brass rod with counterpoise at one end and socket for pen at the other The rod passes through a wooden cross-bar, which is supported on two steel points that rest on a velvet-covered board.

with Goold's pendulum, described later on in this work, but much smaller weights can be used, as the friction of the gimbal mount is much less than that of the ball bearing. By raising the weight

on the upper pendulum the figure is sharpened; by lowering it is flattened. Thus 1 : 3 harmony will require the weight of the upper pendulum to be higher on the rod than is required for 1 : 2, but lower than for 1 : 4. Raising and lowering the lower weight has the same effect as raising and lowering the upper weight, but it affects the harmony less, and is therefore useful as a fine adjustment.

Lightening the weight of the upper pendulum also sharpens the harmony, as does also the increase of the bottom weight, so that there are various alternatives available for regulating the periodicities, but with regard to the method of lightening or increasing the weights it must be remembered that they must not be so disproportioned that one pendulum comes to rest long before the other. This defect in the working of a twin-elliptic is not, however, entirely due to the proportions of the weights, and the causes of it are somewhat obscure. With such a harmony as 3:10, or any harmony of a complex character, it does not occur, but with a 1 : 3 harmony it seems to be unavoidable, and whatever the relative proportions of the weights, this figure soon tends to become a mere ellipse. In the case of the pendulum described in the previous chapter, the initial amplitude of the pendulums is the determining factor in making all come to rest together, because the pendulums are separate, but with this instrument the problem is complicated by their influence upon each other.



: 3. Counter current.

PLATE X.

FINDING THE HARMONIES.

To arrive at harmony, try by moving the upper weight to different positions to get an approximate ratio. If the repeat line falls in advance, the figure wants sharpening by one of the methods described above. If it lags behind, it requires flattening.

The laws of the harmonies of different ratios are, as in the instrument described in the previous chapter, as follows:—

Antagonistic rotation. Cusps are to the outside of the figure, and equal in number to the *sum* of the numbers composing the ratio.

Concurrent rotation. Nodes are within the figure and equal in number to the *difference* of the numbers composing the ratio.

And as was explained on page 54, if the number of nodes (concurrent and antagonistic) are added together, half the sum will be one of the numbers of the ratio, and of course the other is then easily got at.

ORDER OF SUCCESSION OF THE NODES.

The sum of the numbers composing the ratio may be the same in various harmonies. For example, there will be 11 external loops in a 3 : 8 harmony, also in 2 : 9 harmony, and in a 4 : 7 harmony, the sum being 11 in each case. But the order in which the loops are successively formed will not be the same in the three figures. If the loops are designated by the first eleven letters of

the alphabet arranged clock-wise, the order of their formation will be as follows in the three respective cases :—

		a	b	c	d	e	f	g	h	i	j	k
3 : 8 harmony	...	1	9	6	3	11	8	5	2	10	7	4
4 : 7 ,,	...	1	5	9	2	6	10	3	7	11	4	8
2 : 9 ,,	...	1	6	11	5	10	4	9	3	8	2	7

Thus in the 3 : 8 harmony the second loop to be formed will be the eighth point (h) of the 11-pointed star composing the complete figure. In the 4 : 7 harmony the second loop to be formed will be the

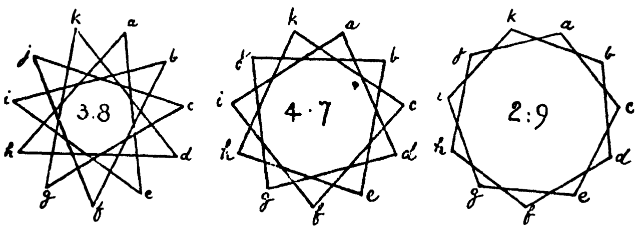


FIG. 11.

fourth point (d) of the star. In the 2 : 9 harmony it will be the tenth point (j). The effect of these various formations on the character of the figure is obviously that with the increase of the acuteness of the angle at the points the central polygonal space is diminished, and vice versa. Thus the 2 : 9 harmony makes very wide angled points and describes a proportionately large central undecagon, while the 3 : 8 harmony involves a smaller central undecagon with more extended acute angled points. The ratios 1 : 2, 1 : 3, 1 : 4, &c., represent extreme

cases in which the triangle, parallelogram, or pentagon compose the whole figure. It is in these figures, and those that approximate them, that the antagonistic phase most readily lapses to the unison spiral as the figure proceeds. On the other hand the figures which for the reasons explained above have the acutest initial points retain the antagonistic phase longest.

REGISTERING THE HARMONIC RATIOS.

Workers would save themselves much loss of time and trouble if they would be at the pains at the outset to register the positions of the weights for the principal harmonies. The instrument should easily give a compass of ratios from 1 : 2 up to 1 : 4. The following table shows the series in order upwards, omitting ratios of large numbers :

$$\tfrac{5}{9} \quad \tfrac{1}{2} \quad \tfrac{4}{9} \quad \tfrac{3}{7} \quad \tfrac{2}{5} \quad \tfrac{3}{8} \quad \tfrac{1}{3} \quad \tfrac{3}{10} \quad \tfrac{2}{7} \quad \tfrac{1}{4} \quad \tfrac{2}{9}$$

Of these the only two symmetrical harmonies are $\tfrac{3}{7}$ and $\tfrac{1}{3}$.

The following table appertaining to the writer's own instrument may be useful as showing the form of register required, and may also help in arriving at the approximate positions of the weights for an instrument of about similar size. Length of pendulum from table top to knife edge, 12 ins. From knife edge to ferrule, 18ins.

Table A. Higher Harmonies.

Fixed conditions :—

Upper weight, 4lb.

Lower weight, 3lb.

Lower weight at a distance of 18½ inches from the point of suspension.

	Distance of collar of upper weight from knife edges. Inches.	Harmony.
1	16¾	2 : 5
2	15¼	3 : 8
3	12¾	1 : 3*
4	10⅞	3 : 10
5	10	2 : 7
6	8⅛	1 : 4
7	6¼	2 : 9

To lower the scale so as to get in the rest of the harmonies enumerated above, the lower weight must be reduced. Making this only 1lb., the other conditions being as above, the remaining 4 harmonies are as follows :—

Table B. Lower harmonies, $\frac{5}{9}$ to $\frac{3}{7}$.

	Upper weight from knife edges. Inches.	Harmony.
8	17	5 : 9*
9	14	1 : 2
10	11	4 : 9
11	10½	3 : 7*

By raising the upper weight higher than 12¼ inches the harmonies given in Table A may, some of them, be reached and the same figure obtained as with the heavier lower weight.

Table C. Lowest harmonies $\frac{3}{4}$ to $\frac{4}{7}$.

Upper weight 12lb.

*Symmetrical.

Lower weight ½lb.

Lower weight 18½in. from point of suspension.

	Inches.				Harmony.
12	...	17	... ...	3	4
13	...	14½	... ...	5	7*
14	...	12½	... ...	2	3
15	...	10¾	... ...	5	8
16	...	10	... ...	3	5*
17	...	9	... ...	4	7

These three arrangements provide thus for the 17 harmonies named, and, of course, for the intermediate ones composed of higher numbers. The ratios are given in scale order, and the ratios marked with an asterisk (*) are symmetrical.

TO SECURE CONCURRENT ROTATION.

Grasping the pendulum rod near the top, a gentle circular movement is imparted to it—an arc of about the third of a complete circle. When released it will take up a movement with internal nodes—the law of the disposition of which has already been given.

TO SECURE ANTAGONISTIC ROTATION.

Grasping the rod again near the top and giving it a rectangular movement (diagonal to the knife edges), succeeded by a second rectangular movement, and then a third, so as to describe in the three movements something like an equilateral triangle, a figure in opposed rotation, with external

cusps, will be produced. At the first attempt there may be a little difficulty in obtaining this triangular starting movement, but after a certain number of trials the hand will gradually learn to accommodate itself to the conatus of the pendulum with the greatest facility.

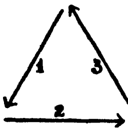


FIG. 12.

The triangular hand movement may then be kept up for some time before releasing the rod, only taking care to obey the natural time and direction-tendency of the pendulum. The knack of starting the instrument in this way seems so unattainable at the first attempt that a beginner may be discouraged, and think it impossible, but it comes with practice, and once acquired it becomes quite instinctive to move the hand in time with the rod. The triangular movement should be very small to begin with, and then continued with progressively increasing amplitude. In this way the amplitude can best be kept uniform in each direction. If it is desired that the cusps shall be rounded, the movement of the hand should be infinitesimally slower than the natural time period of the rod, *i.e.*, the hand should restrain its movements very slightly all the time. A very slight quickening of the movement at each rectangular stimulus will make the cusps pointed, and a little more quickening will convert them into loops. When these principles are thoroughly understood it is only a matter of practice to become

proficient in starting the pendulum so as to produce any desired order of pattern.

WATERED "RADIATION" EFFECTS.

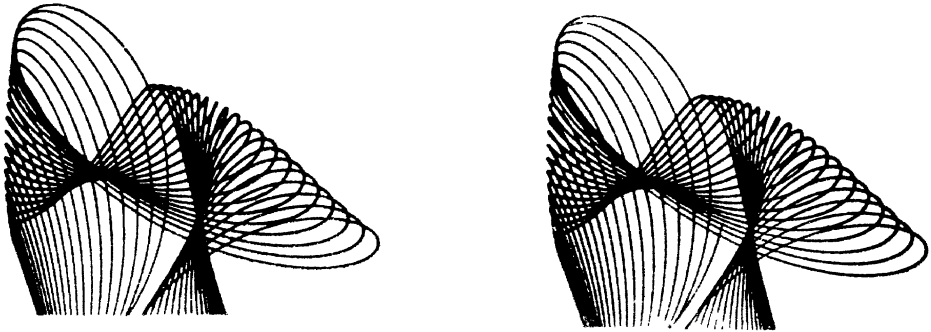
With such ratios as 3 : 5, 3 : 7, 4 : 7, and others, very wonderful effects of watering are obtained by starting the instrument so as to allow the lower pendulum only very slight separate movement. As the pattern proceeds the intersecting lines form if the harmony is perfectly adjusted, and the figure has been started as nearly as possible circular, radiating bands equal in number to the sum of the numbers composing the ratio. These radiations curve over in the direction of the rotation, if the harmony is slightly flattened, and in the opposite direction, if it is slightly sharp—a fine test for the exactness of a harmony.

STEREOSCOPIC FIGURES WITH THE TWIN-ELLIPTIC.

It has already been shown how stereoscopic effects may be produced with the ordinary harmonograph, by making a pair of figures of identical size, but differing in phase, both phase and size being under control, when the movements of the pendulums are rectangular. But how is the stereoscopic effect to be obtained with the figures of the twin-elliptic pendulum, in which it is obvious that it is impossible to secure this control over either the size of the figures or their phase? The problem is generally assumed to be an insoluble one, and

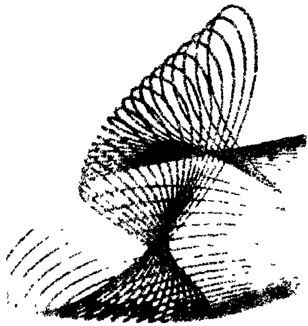
though it is evident that the shell-like forms of the twin-elliptic figures would produce an immense variety of very interesting forms, if they could be stereoscopically rendered, it is usually assumed to be quite impossible to do so. There is a way to do it, however, and the method is very simple, while the results are even more effective than those obtained with the stereoscopic rectangular figures, because the twin-elliptic curves can be carried much further for stereoscopic tracings, the effect in the stereoscope being much enhanced. The principle on which the stereoscopic pair is produced is as before, by the introduction of a slight difference of phase, but in this case that difference is brought about automatically in a very interesting manner.

In order to explain both the principle and the method, it must be pointed out at the outset that in any figure which is only an approximate harmony a continual change of phase is going on all the time that the curve is being produced. It is this gradual change that gives so much of the grace and beauty of the figures, which are, nearly all of them, only approximations to complete harmony, the change of phase occurring so gradually, as the figure proceeds, that symmetry is preserved and variety is introduced at the same time. This being premised, it will be understood that in a symmetrical figure, such as an approximate 1 : 3 harmony, the opposite parts of the figure, which appear to the eye to correspond, do not match exactly, but differ by the



Inverted Figure — Showing Stereoscopic Effect

PLATE XII

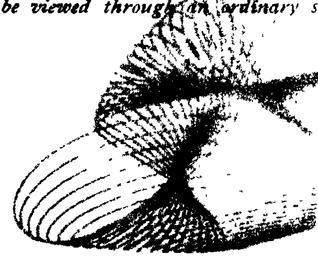


BISECTED TWIN-ELLIPTIC SYMMETRICAL FIGURE,

showing that the two halves are not identical, but that the progressive change of phase of the figure asserts itself sufficiently to give stereoscopic effect to the two halves.

See page 71.

To be viewed through an ordinary stereoscope.



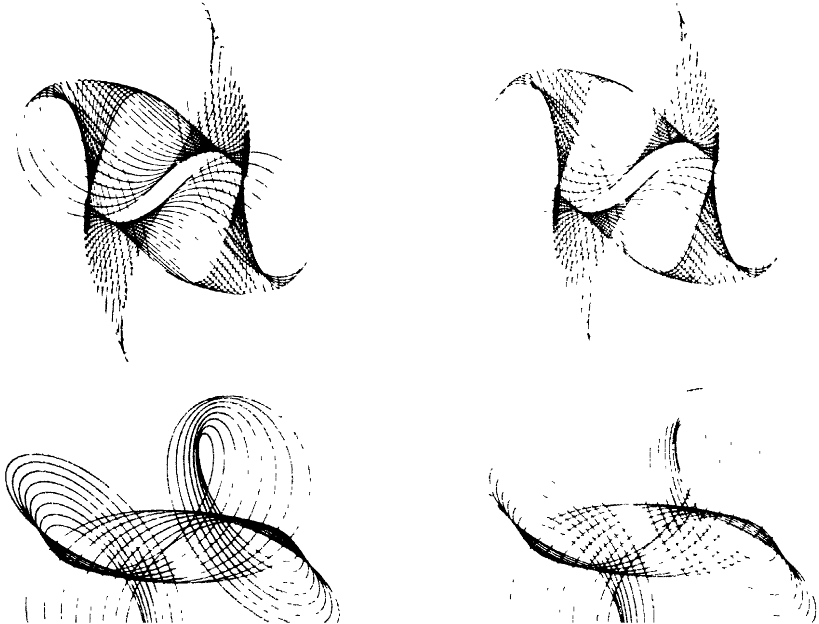
infinitesimal change of phase which had crept into the curve in the interval between the tracing of each portion of the curve on the right and its corresponding equivalent on the left. If we cut a figure into two halves, though the two portions seem to the eye alike, a closer inspection will show minute differences, which are due, not to any imperfection of the instrument, but simply to the fact that the figure is changing phase as it proceeds, and that, therefore, a variation creeps into the opposite corresponding portions of the curve throughout. The differences, though slight when the curve is quite close to absolute harmony, are yet sufficiently marked to produce strong stereoscopic effect, and if the two halves of the bisected figure are placed side by side in the stereoscope, it will at once be seen that the result is a beautiful solid curve. (See Plate XII.)

After making this interesting experiment, it will be easy to understand that if we had two photographic copies of one of these curve tracings, there would be no need to bisect the figure to get stereoscopic effect, for all that would be necessary would be to turn one of them up-side-down. The two being then placed side by side in the stereoscope, it must follow that what was originally the left hand portion of the curve is made to combine with the right hand portion, and as the two sides correspond except for the slight variation of phase, the stereoscopic result will be attained.

The photo reproductions may be obtained by smoked glass as already explained on pages 43 and 44, and the greatest care must be taken so to start the figure that when one of the prints is inverted it shall not present any lines without an equivalent on the other side. This is only effected by making *the starting and finishing points at inconspicuous places, such as at points of intersection.* It needs a *little practice, not to say luck, to do this quite perfectly.*

If preferred, a curve traced in ink on paper may be photographed for the negative. It is best to use black ink for this purpose. Having made the two prints from the negative, rule a square margin round one of them with pencil, and having duly inverted the other, adjust it, still inverted, behind the other, so that when the two are viewed in front of a strong light both prints coincide as nearly as may be. Holding the pair firmly in this position, cut them both out together along the lines ruled round the top one, and the two are then ready to mount side by side.

A simpler way to make a negative must also be described, and this useful process applies not only to the making of negatives for this particular purpose, but equally for reproducing any of the harmonograph curves. The plan is simply to trace the curve with photo developing solution instead of ink on the sensitised surface of a photographic film, such as a Frena film. Sensitised plates do not



3

Twin Elliptic Stereoscopic

Each pair is a reproduction of the same figure, but one copy is inverted

PLATE XIII.

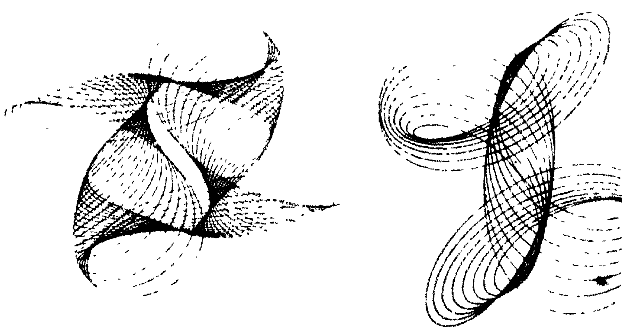
Plate XIII.

TWIN-ELLIPTIC-STEREOSCOPIC FIGURES PRODUCED BY INVERSION METHOD.

Harmony 1 : 3.

See page 71.

To be viewed through a stereoscope.



answer so well, as the surface is hard and unsuited to the glass pen. Of course, no dark room is required, and the work can be done in subdued daylight. All that is requisite is to fix the film, in the ordinary photograph fixing solution, after the tracing has been made and has been exposed to a good light for a few moments. The result is a negative which will give white line prints beautifully suited for stereoscopic effect. The developer should be fairly strong, and it is well not to expose the films to an unnecessarily bright light before or after the tracing, as solarisation may ensue, preventing the proper action of the developer.

It must, of course, be understood that this method of rendering the twin-elliptic figures stereoscopic only applies to those of the curves that are symmetrical harmonies, that is, to those that are composed of ratios that total to an even number, as explained on page 55. The others, being unsymmetrical, cannot be inverted so as still to present corresponding loops and nodes. The most effective figures are those drawn with the pendulums in antagonistic rotation.

The further removed from actual harmony a figure is, the greater will be the difference of phase between its opposite sides, and therefore the more marked will be the stereoscopic effect, but this must not be carried too far, for at the same time the general symmetry of the figure tends to be lost.

HEKTOGRAPH STEREO FIGURES.

A simple way to produce the stereoscopic forms is by the use of an ordinary hektograph—the well-known copying process that utilises a tray of gelatine softened with glycerine. The patterns should be drawn on a good cream-wove paper with a strong ink, such as violet *copying* ink. The tracing is laid face downwards on the gelatine and left about two minutes. When the paper is pulled off, the tracing will be seen on the gelatine surface. A piece of white paper is then put down on this and the pattern printed off on it, leaving the paper in contact for at least two minutes, so as to secure a dark impression. At least three pairs of stereo figures can often be secured from one tracing. After that they may begin to get a little faint. The lines reproduce perfectly, and if the printing is well done they look exactly as if they had been traced with the pen in coloured ink. With some inks the colour is strengthened by warming the finished prints at the fire, and if the lines are bluish they look all the more intense if printed on a yellowish paper.

The hektograph is by no means to be despised for reproducing any of the curve tracings—even those with the finest "waterings." The original copy is not spoilt in the process, and its intensity is rather strengthened.

STEREOSCOPIC FIGURES WITH TWO PENS.

Instead of reproducing the figure for the two copies the pair may be traced by using 2 pens

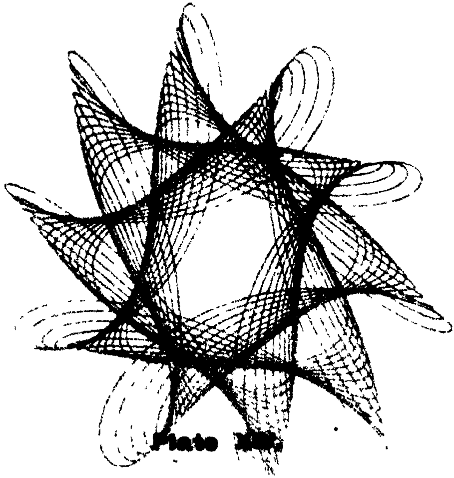


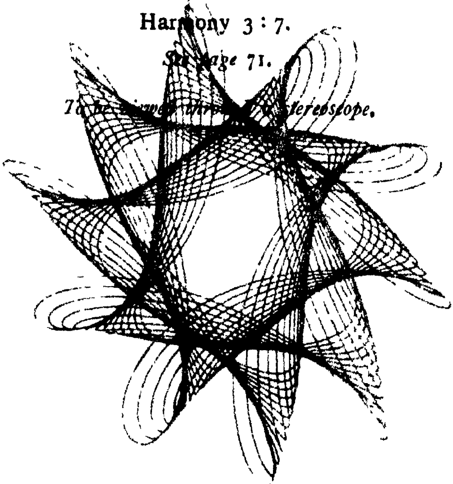
Plate [illegible].

TWIN-ELLIPTIC STEREOSCOPIC FIGURES PRODUCED BY INVERSION METHOD.

Harmony 3 : 7.

See page 71.

To be viewed through the stereoscope.



37. Counter current stereoscopic (inversion method)

with separate levers, each as the one shown in Fig. 10. The chief difficulty is to drop and lift them together. One way of starting them at the same moment is to lay a loose sheet of thin smooth paper on the paper that is to receive the tracings, and having started the tracings on this, draw it quickly but gently away in such a direction that the two pens touch the paper below at the same instant.

HOW TO CENTRE THE PAIR OF FIGURES PERFECTLY.

The tracing of the stereoscopic figures with two pens in this manner is much the simplest and easiest way of producing them, and by adopting a further device for centreing the pair the inversion is more satisfactorily accomplished, and the necessary parallelism ensured for the inverted pair. To do this let the paper that is used to overlay the tracings be 5 inches by 2½ inches, and divide it by a central line into 2 squares each 2½ inches, making a central dot in each. Adjust the pens so that when at rest they are on these central points and before starting the instrument indicate on the underlying paper (which is to receive the tracing) the four corners of the squares, making a dot to show where they fall. After the tracing is made join up these dots and cut along the lines, when two exactly centred figures will result each on a square 2½ inches side. Specimens of twin-elliptic stereo figures will be found in Plates XIII. and XIV.

ANOTHER FORM OF BENHAM'S TWIN ELLIPTIC.

The curves may be equally well traced by a pen attached to the pendulum top, the paper being on a fixed board at an appropriate level. In some ways this arrangement is preferable, especially for the stereo patterns just described. The square board at the top of the pendulum is replaced by a wooden strip to take the points of the pen lever. The chief advantages of this arrangement are that there is not so much up and down movement of the pen, while the formation of the curve can be watched much more comfortably, especially in the case of harmonies involving rapid movement. Moreover, the harmony is not affected by the weight of the glass if smoked glass is substituted for paper. The wooden strip for the pen lever should be long enough to take two pens, for this arrangement is particularly well suited for the stereoscopic figures.

VI. SYMPATHETIC OSCILLATIONS.

INTERESTING effects are produced by describing a unison spiral, while at the same time and working through a hole in the same table a similar gimbal-mounted pendulum is swinging quite unattached in any way to the one which is used for the tracings. Although the only connection between the two pendulums is by the medium of the table itself, the oscillations of the free pendulum

will record themselves in the form of alternate widenings and approximations of the lines of the spiral which will thus be banded, the frequency of the bandings depending upon the ratio of the two systems of oscillation. It will be observed that the bandings, when the two pendulums are concurrent, will take the form of concentric rings. When the pendulums rotate in antagonistic directions the banding forms a continuous spiral. Specimens are given in Plate XV.

VII.—GRAPHIC REPRESENTATION OF HARMONOGRAPH CURVES.

A PENDULUM swinging in the circular orbit A B C D E F (Fig. 13) accomplishes its round in exactly the same time as it would take to

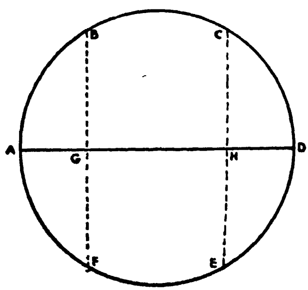


FIG. 13.

swing in a straight line across the diameter and back again, *i.e.*, from A to D and from D to A. Similarly it will accomplish half its round—A B C D—in the

same time as it would take to swing in a straight line from A to D; or again one-sixth of its round A B in the time it would take to swing in a straight line from A to G. But from B to C is also a sixth of the circle, and its equivalent in the straight-line is G H. Neither A G nor G H are a sixth part of the straight line orbit, so that it is evident that while the circular movement is of uniform velocity the rate of oscillation varies along the straight line path, each of the six divisions A G, G H, H D, D H, H G, and G A being traversed in exactly the same time, viz., $\frac{1}{6}$ of the time taken by the whole oscillation to and fro, though the length of the intervals varies, showing that the pendulum travels faster in the middle part of its orbit and more slowly towards the ends.

By dividing a circle into a larger number of parts, say 24, and ruling lines vertically and horizontally through the divisions the paper will be divided into a series of divisions, vertically and horizontally, each one of which would be traversed in one 24th part of the time taken to traverse the whole line. In other words the series of rectangles formed are *squares* as regards *time*, however much they vary as regards *space*, and by means of these *time-squares* the curve can be graphically represented for any two compounded pendulum movements, whether swinging in straight lines, in ellipses or in circles. (Fig. 14.)

To plot the curve for *rectilinear movement* number

the horizontal lines from 1 to 24 and also the vertical lines from 1 to 24. If for a unison figure, number each line; if for a 1 : 2 harmony, number each vertical line, and every second horizontal line. For a 1 : 3 each vertical line and every third horizontal line and so on. For a 2 : 3 harmony every second vertical line and every third horizontal

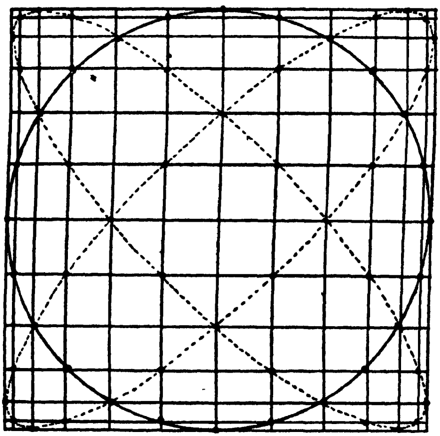


FIG. 14. "SQUARES" OF EQUAL TIME ON PENDULUM MOVEMENT.

line, and so forth according to the ratio. The different *phases* depend upon the *starting point* of the two series of numbers. Having numbered the lines according to the ratio and phase required, place a dot at the point of intersection of each pair of lines of similar number. Join up the dots

and the curve of the compound movement is given. (*Fig.* 15.)

For compounded *twin circular movement* number the 24 divisions of the circle shown on the "time-square" paper. Number the points inside the circle and again outside, the two series of numbers being

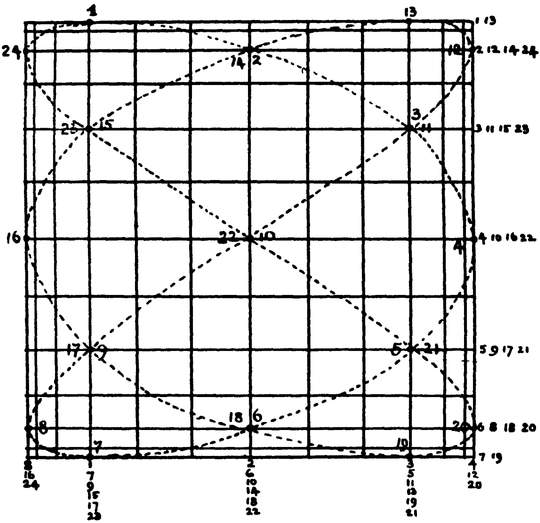


FIG. 15. RECTILINEAR 2 : 3 HARMONY CURVE AS PLOTTED BY MEANS OF "TIME" SQUARES.

again made in accordance with the ratio and phase of the harmony, and the order of the two sets of numbers being in the *same* direction round the circle for *concurrent* motion and in *opposite* direction for *antagonistic* motion. In the case of circular move-

ment the squared paper does not give intersection points for the pairs of numbers, but the determining points are found by *bisecting the line joining the pairs.* Lay a straight edge between the pair of points and with a pair of dividers find the centre of the line formed by the straight edge. Place a dot at this

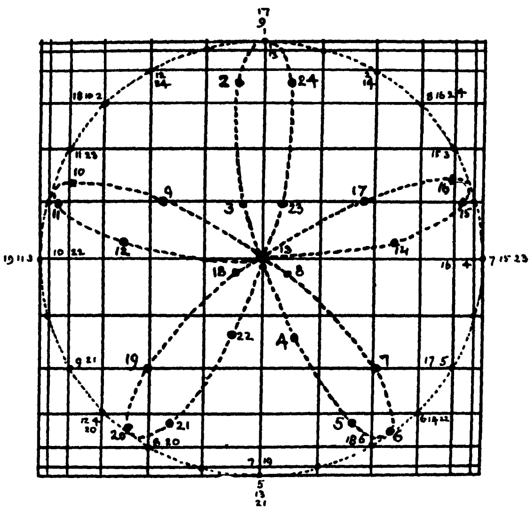


FIG. 16. 2 : 3 TWIN CIRCULAR PLOTTED BY MEANS OF "TIME" SQUARES.

point and similarly bisect the line joining the next pair of numbers. Join up these dots and the curve is given. (Fig. 16).

For compounded *elliptical movement* proceed in the same way, only instead of the circle, take the two *ellipses* shown on the "time-square" paper. The

distance between any two of the 24 divisions of these ellipses represents an interval of equal time, for in every case it is a *diagonal* of a "time-square," and therefore in point of time they are all equal. (Fig. 17.)

Compound-movement curves of *straight lines at less than a right angle*, of *circular-cum-elliptical* or

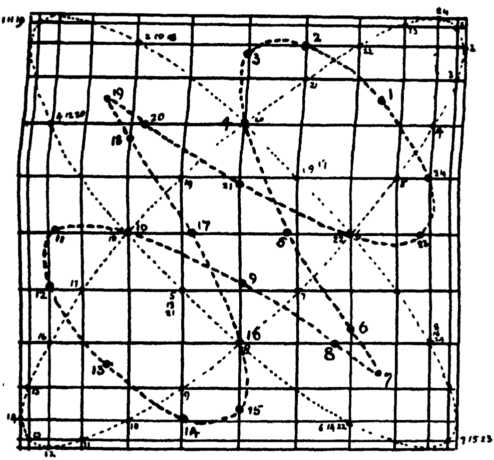


FIG. 17. 1 : 3 OPPOSED TWIN-ELLIPTIC PLOTTED BY MEANS OF "TIME" SQUARES.

circular-cum-rectilinear oscillation, are all capable of representation by means of the squared paper and the method of bisection. It will be noticed in making the tracings that in the case of circular movement changes of *phase* do *not alter the character of the figure*, but only shift its *position*.

The varieties of curve produced by varying the *amplitude* of the respective movements are represented by altering the scale of the divisions of equal time. This is most easily effected in the case of the circular movement. It is only necessary to make *two concentric circles*, each divided into 24 equal parts, and with their respective diameters in the proportions of amplitude required.

Experiments will show that the following interesting law as to amplitude prevails:—

If the amplitudes of two *circular* movements are in *direct proportion* to the ratio, the nodes form *points*. If they are in *inverse proportion*, the nodes are *arcs of circles*. Thus a 2 : 3 harmony of circles in opposed rotation, 2 inches and 3 inches in diameter respectively, will give a perfect *pentagram*. But if the numbering on the circles is transposed, the numbers on the smaller circle being transferred to the larger one and vice versa, the five nodes are arcs of circles. If the ratio of amplitudes is *higher* than that of the period, the *points* become *loops*, and if this arrangement is tried inversely, the *arcs* become *elliptical*.

PETAL-FORM CURVES.

The curves resulting from compounding unison movements at right angles with arcs substituted for straight lines are particularly interesting, because they take the shapes of flower petals, different phases giving varied forms, from the broad cordate shape of the buttercup petal to the acuminate form in the daisy. Inasmuch as the

flower stems sway in arcs, these forms are suggestive as being possibly connected with petal development. The following are a few of the forms :—

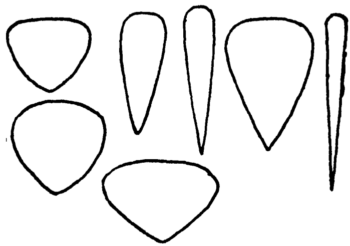


FIG. 18. PETAL-SHAPED CURVES—COMPOUNDED ARC UNISON MOVEMENTS.

It is evident that a flower stalk is a pendulum which, as it sways in the wind, executes compound vibration curves of the character described, and it seems not at all impossible that these natural movements are a factor which must be taken into consideration in studying the evolution of leaf and flower forms, or certainly as a factor in the unfolding of buds. The following interesting experiment is highly suggestive in this connection. Attach *firmly* to the end of a thin steel rod (such as a hat pin or knitting needle) a tightly closed flower bud. Secure the wire at the other end firmly in a vice and set it vibrating with as wide an amplitude as possible. In a few moments the series of rapid vibrations, repeated three or four times, will have shaken the bud into a fully opened flower. Further experiments in this direction might be productive of very important results.

PART II.

VIBRATION-FIGURES: THEIR PRODUCTION & CONSTITUTION IN NATURE AND IN THE WORKSHOP.

By JOSEPH GOOLD.

VIBRATION-FIGURES.

(I.)—INTRODUCTORY.

VIBRATION figures are the paths marked out by the movements of vibrating points, particles, atoms, molecules or "bodies," of any magnitudes, velocities, or complexities whatever.

The figure marked out by a *single* vibration is a straight line; all other vibration-figures are, therefore, figures of the combined movements of two or more vibrations; hence they are commonly called "compound vibration-figures." I prefer, however, to use the shorter term in the comprehensive sense of the longer; for all vibration-figures are compounded from simple vibration-elements, and the straight line itself may be the resultant of any number of vibrations. Let it therefore be understood that the figure of a single vibration must be straight line; but a vibration-figure—whether a straight line or not—may be any combination of straight lines.

The forms of vibration-figures depend entirely upon comparative conditions relating to amplitudes, rates, and phases of constituent vibrations, and are completely independent of actual dimensions or velocities. It is necessary to grasp this fact most thoroughly, as a right apprehension of its significance will vastly simplify the general consideration of the subject and will facilitate the formation of correct ideas about the relation of vibration-figures to the phenomena around us.

In the following pages we have to speak about things that demand accurate thinking, and it may assist our purpose to begin with a definition. But let me first warn the student that definitions are necessarily more or less untrue, because they attempt to circumscribe the infinite. Moreover, the subject is inexhaustible, and my chief aim must be to help the reader to think about it.

With this proviso I may define a vibration as *the motion of a fixed object.*

It will doubtless be observed that this is both illogical and contradictory. Yes; but logic, like many other good servants, is a bad master; and although "the letter killeth—the spirit giveth life." Now the spirit, or interior meaning, of above definition is this: that all vibrating bodies are *in some way* fixed. A few examples will make this clear.

1. Our earth and all the heavenly bodies, whilst freely vibrating in their orbits, are fixed to their respective systems by gravitation.

2. Every sonorous particle in the atmosphere—so far as its acoustic properties are concerned—is fixed in its own position by the balanced pressure of its surrounding particles.

3. Every pendulum is fixed to the centre of its own field of action by means of its suspension-point.

4. So with every vibrating bar, plate, or tuning-fork; the vibrating portion must be fixed to the "nodal" portion, or no vibration can take place.

5. So again with piano-wires, violin-strings, drum-heads, etc. Some sort of framework supplies the indispensable fixing-points from which the various forms of vibration originate.

FUNCTION OF THE FULCRUM.

From such facts as these it is easy to perceive that the chief essential in any vibration system is the way in which it is fixed. This may be described in one word as the fulcrum. It will, perhaps, serve to make evident the unity and universality of vibration phenomena, if we try to think out how the fulcrum affects the character of the system in a few particular cases.

1. *The planets and heavenly bodies generally.* Here the fulcrum is the balance of gravitation from surrounding bodies. In most cases this becomes a balance between direct gravitation and inertia, acting at right angles to each other—as with our earth, for example. Gravitation alone would pull the earth straight to the sun ; inertia alone would carry the earth away at a tangent, exactly at right angles to the gravitation line. The two forces *together* cannot work for a moment in either of these two lines, but must necessarily adopt a line between them, which is no other than the earth's orbit. The distinguishing feature of this method of working is the complete absence of friction, and the direct consequence is that the full amplitude of the vibration remains unchanged. The vibrating body

(the earth) moves repeatedly and perpetually over the same line, for there is no consumption of energy ; the elasticity of the system is perfect . . . . when disturbing circumstances, such as the action of the tides, are left out of the calculation.

2. *Air-particles.* In the vibration of an air-particle the fulcrum of the system is the elastic mass of the surrounding medium. The duration of the vibration ceases with the quiescence of the disturbing force, because the vibrating body has but the least possible momentum ; the vibrations are all "forced"; their energy is derived from an external source, and the chief characteristic of the system is mobility. The consequence is that this class of vibration is susceptible, to an almost unlimited extent, of continuous transformation ; its mutability is unparalleled.

3. *Pendulum.* The fulcrum of the pendulum is the point of suspension. Here originates its whole action, accompanied by an equal and opposite reaction. Hence the necessity for fixing this point with the utmost possible firmness, because any infirmity in the structure supporting this point means a cumulative inequality in the dissipation of energy of the system, and that means the gradual elimination of one of the constituent vibrations of the ellipse, ending in the solitary survival of the straight line. This is the most common of all pendulum defects; and, in my opinion, it is so

difficult to eradicate that not even the elaborate arrangements that are sometimes employed in connection with Foucault's experiment, for showing the rotation of the earth, are sufficient to make the results altogether trustworthy. The essential movements of a twin-elliptic pendulum resemble very closely the orbital motions of the earth and other celestial bodies. The earth and moon may be regarded as a great twin-elliptic pendulum swinging from the sun with a ratio of 13 : 1 "concurrent." The resulting deflections in the earth's orbit are, however, very small in comparison with those of ordinary pendulum figures, as the ratio of the orbits, $\frac{\text{moon}}{\text{earth}}$, is only about $\frac{1}{360}$ and the relative masses about $\frac{1}{80}$; so that, in effect, these disturbances amount to little more than those which we recognise as the tides. The character of the stellar movements is stability, because the distances between fulcrum and centre of oscillation are comparatively fixed; whereas in the pendulum these distances are alterable at will; the character of its action is, therefore, adaptability.

4. *Tuning-fork.* The fulcrum of the tuning-fork is, in a pre-eminent degree, self-contained. On this depends its special freedom from friction, and hence the durability of its vibratory action. It acts most freely in a vertical position; but this is only noticeable when the mass becomes considerable. The audibility of the tuning-fork depends upon the mass-motion of its limbs being trans-

muted into molecular motion by contact of its stem-point with some resonant body. The particles of *the resonant body must be in such a condition that they readily respond to the fork-rate of vibration and* to simple multiples of the same. Hence the actual sound of the tuning-fork consists much more largely of "harmonics" than is commonly supposed. The chief characteristics of the tuning-fork are the prominence and persistence of its fundamental vibration and its marvellous constancy of pitch. A straight steel bar of similar dimensions is, however, equally constant in pitch and, when properly mounted, nearly as persistent in the maintenance of its findamental vibration.

5. *Bars, plates, etc.* When these are free at the ends the fulcrum or fulcra may be at any of the nodal points*; and if the bar were supported by floating in a fluid of its own specific gravity, its fulcrum would be identical with its nodal system. Practically, therefore, the fulcra of the plate are the node-rests employed for its support, and in many cases it makes little difference at what points in the system these rest sare applied. But when the plate has two or more systems of vibration of nearly similar pitch the adjustment of the node-rests becomes a matter of the first importance. In these cases everything depends upon the comparative facilities afforded for the development of

* "Nodal points" may be more accurately described as "Reversion points." They are the points where the direction of motion is reversed.

the respective systems, and nothing affects those facilities more intimately than such adjustment. That system is most facilitated which has its nodal lines most nearly in position with the rests; and that is most obstructed which has its anti-nodes, or "ventral segments," in this position.

When the pitch-difference between two systems is extremely small (as in the case of "interfluxion plates") it is only by very careful arrangements of the node-rests that either system, separately, can be rendered tolerably stable.

Such a condition may be described as one of versable stability.

6. *Strings, films, membranes, etc.* Bodies of this class vibrate by reason of the tension applied to them by means of a suitable framework, and the fulcrum of the system, in any case, is the point of juncture between the vibrating body and the framework supporting it. In pianos, violins, etc., the effective part of the string ends at the bridge; in such cases, therefore, the bridge is the acoustic fulcrum. As an illustration of the large importance of precise adjustment of the fulcrum under special conditions, I may mention the case of a steel plate which I exhibited at the Royal Society's Conversazione on May 8, 1907. This plate measured $33 \times 4 \times \frac{3}{8}$. The rates of vibration for the three systems i. lateral, ii. dual, and iv. normal were 724, 719, and 650 respectively; whilst that of the fundamental note (i. normal) was very nearly

identical with the "difference-tone" between i. lateral and iv. normal— that is, 724 — 650 = 74.

When the 2 node-rests were placed under the 1st "iv" nodes each side of the centre, and the plate was excited by applying the "generator" to the anti-nodal region of i. lateral (that is, on the middle of the edge of the plate) nothing was at first observable but the ordinary vortex action between i.L and ii.D. But when the excitation was increased beyond a moderate intensity, there was seen a most extraordinary phenomenon. The nodal line between the vortices suddenly wavered and then broke into a tumultuous eruption, accompanied by a low rumbling sound; and as the tumult subsided the node-formation became completely transformed into the system proper to iv. normal—a perfect nodal revolution! But the most remarkable feature of this strange occurrence was that it happened only when node-rests were used which were fitted with 1in. long india-rubber strips. With rests of a different form, which answered perfectly for all ordinary purposes, nothing of the kind could be made to happen. This could only mean that the difference in the node-rests made all the necessary difference in the balance of facilities requisite for the production of the varied phenomena.

(II.)—VIBRATION-FIGURES AND THE TWIN-ELLIPTIC PENDULUM.

VIBRATION-FIGURES, as we have just seen, are the forms of motion produced by the combination of two or more vibrations. But what is a vibration ? If you knock or shake an elastic body—such as a cane, a piece of glass or metal, etc.—you will hear, see, or feel a vibration or vibrations, according to the circumstances. If you shake an inelastic body, such as a piece of wax or lead, or a bag of flour, no vibration will be perceived. In the case of the inelastic body the imparted blow produces a permanent alteration of its position, shape, or condition ; with the elastic body, on the other hand, there is no permanent alteration, but a state of vibration is set up—the body moves regularly to and fro (in one or more directions) and gradually comes again to rest. Now, each one of these to and fro movements is a "vibration," and as vibration-figures are combinations of such vibrations, we must look a little more closely at the single vibration in order to gain an intelligent idea of its possible combinations. Note first, that when we have to be precise we speak of a single vibration as a "harmonic vibration." A mere "vibration," in the ordinary sense, is not precisely definable ; but a "harmonic vibration" is to and fro motion in a straight line, in which the velocity of the moving body is always changing

from zero at the ends to zenith at the centre ; and the rate of change may be shown as follows :—

Consider the vibration-path to be the diameter of a circle A.B. (Fig 19.) Then, if the semi-circle

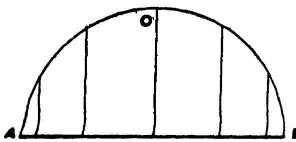


FIG. 19.

be divided into equal parts (any number) and straight lines be drawn from the points of division perpendicular to the diameter, the divisions of the diameter so produced will indicate the points passed over by the moving body in equal time periods.

This definition enables us to calculate and trace out the forms produced by many complex combinations of vibrations, as will be shown later on.

Vibration figures may be conveniently considered under three classes—1st, those that are wholly natural in their origin ; 2nd, those that are wholly artificial ; 3rd, those that are partly natural and partly artificial.

Natural vibration figures as a rule are either too large or too small, too quick or too slow, to be readily discernible, but some familiar forms are open to common observation, such as ocean-waves and ripples, the movements of trees, twigs, cornstalks, and elastic rods of various kinds, rockets, cricket-balls, water-jets, and other projectiles. Even those who

have not had the privilege of witnessing the gambols of the sea may yet have learnt something of the mysteries of liquid vibration from the antics of a tea-cup.

It is not my purpose to enter into a close examination of these interesting phenomena, but I do wish to impress upon my readers the significant fact that vibration-figures are always and everywhere present with us, not only in forms that may be heard and seen and felt, but in many, if not in all, of those wonderful operations of nature which completely defy analysis by our unaided senses.

These more obscure forms of vibration make themselves evident under suitable conditions—as wireless telegraphy, heat, light, actinism, X-rays, etc. Their velocities (more precisely, their "periods") range from thousands to thousands of billions per second ; we can fix no limit for them. But, even so, these only represent one half of nature's vast array of vibration-phenomena ; the scale extends just as illimitably in the opposite direction—in the orbits of the denizens of the heavens. The annual movement of our own earth, for example, is just as truly a vibration-figure as are the circles formed by rain-drops on a water-pool, or the movements of a vibrating rod. In short, the fact must be grasped that the forms of vibration-figures have nothing whatever to do either with any absolute rate of motion in the vibrating body, or with any absolute scale of dimensions.

If our earth, for example, moved millions of times faster or slower than it actually does, and if the magnitude of its motions were altered to any scale, the *form* of its orbit would remain the same. So that, whether or not we are able to interpret them, there can be no doubt that pendulum figures (which are perhaps the simplest natural figures) are most intimately related to the common facts of nature, and especially to music. Not to the art, musical art is at present oblivious of them; not to the science, musical science has hardly yet commenced its existence—but it is quite clear that the principal vibration-figures form the natural basis of all musical phenomena. The two things are in fact essentially identical; both the principal vibration figures and the principal musical intervals are combinations of vibrations in the simplest ratios. In fact, the whole unlimited field of musical sounds (so far as represented by written music) consists of nothing else than vibrations in ratios of the various powers of 2, 3, and 5. Take for example the authenticated chromatic scale of C, which may be thus represented.

Authentic Chromatic Scale of C.

Scale Names:	C	D♭	D	E♭	E	F	F♯	G	A♭	A	B♭	B	C^1
Vibr. ratios:	1	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{6}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{45}{32}$	$\frac{3}{2}$	$\frac{8}{5}$	$\frac{5}{3}$	$\frac{9}{5}$	$\frac{15}{8}$	$\frac{2}{1}$
Powers of 2, 3, 5:	2^0	$\frac{2^4}{3\cdot5}$	$\frac{3^2}{2^3}$	$\frac{2\cdot3}{5}$	$\frac{5}{2^2}$	$\frac{2^2}{3}$	$\frac{3^2\cdot5}{2^5}$	$\frac{3}{2}$	$\frac{2^3}{5}$	$\frac{5}{3}$	$\frac{3^2}{5}$	$\frac{3\cdot5}{2^3}$	2^1

It has already been pointed out that vibration forms are completely independent of "pitch," but the above ratios, if taken at about ten octaves below

ordinary pitch, are seen to be identical with the rate movements of a common pendulum.

So far I have referred chiefly to vibrations and vibration-figures which are entirely natural in their origin, such as waves, ripples, orbits, projectiles, etc., visible and invisible. Our chief concern, however, is with that very large class of figures which is partly artificial and partly natural, being due to the freeest possible operation of natural forces under specially designed conditions. But a few words must first be said about those that are produced entirely by means of artificial contrivances.

(III.) ARTIFICIAL VIBRATION-FIGURES.

ARTIFICIAL vibration-figures are very useful adjuncts in connection with natural phenomena to facilitate the study of compound vibration; they may be produced either by means of specially-designed mechanisms, or by the hand alone, guided of course in either case by intelligent rules and principles. Let us consider the latter first.

DIAGRAM FIGURES.

The simplest case of all is, of course, the single vibration (or any number of such vibrations acting simultaneously), the form or "figure" of which is nothing more than a straight line; but as we have already seen, it is a straight line with ever-varying

rates of motion. This fact must never be forgotten; but yet, to remember it and all that it involves is beyond the capacity of most people. Let us, therefore, in the first instance consider the vibrations we are dealing with to have *constant* motion—to and fro motion, in whatever periods we may require, but with unchangeable rates throughout. This is probably impossible in nature; but the restriction will enable us to deal with our difficulties separately and the method will furnish us with results which may be regarded as skeleton forms of true harmonic combinations.

CONSTANT MOTION FIGURES.

Suppose a tracing-point to move to and fro at a fixed rate in the directions A, B, and C, D simultaneously (Fig. 20). It is obviously impossible for

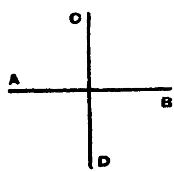


FIG. 20.

a point to move in two such *lines* simultaneously, because it cannot be in two places at once; but there is no difficulty in making it move at the same time in the two *directions*; and the *effect* is the same as it would be if the point moved in the two lines alternately with *infinitely small steps*. This effect is called "the resultant."

Thus, the resultant of combination of the two movements (Fig. 21) A, B, and A, C alternately is a *zig-zag* such as A, D, (Fig. 22) which, with

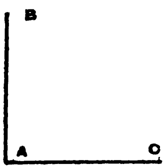


FIG. 21.

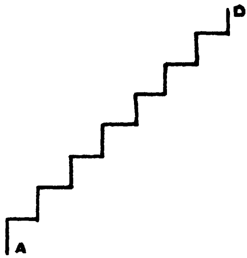


FIG. 22.

infinitely small zig-zags becomes the diagonal line A, D. (Fig. 23), so that when the two to and fro motions A, B, and C, D (Fig. 24) start together from the centre the resultant is to and fro motion in the diagonal E, F (Fig. 25,)

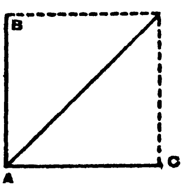


Fig. 23.

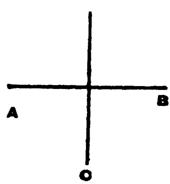


Fig. 24.

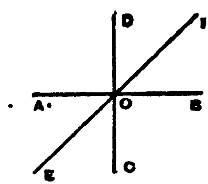


Fig. 25.

This is one of the extreme "phases" of the combination. But now suppose the combination to take place when the single motion C, D has already gone as far as D. The return motion

D, O acting alone would take the point to O, whilst the A, B motion, starting from D, would take it to F. The resultant of these two acting together is D, B. (Fig. 26). At this point A, B begins to return in the direction B, O, whilst D, C. continues

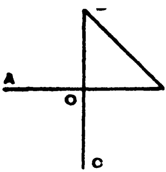


FIG. 26.

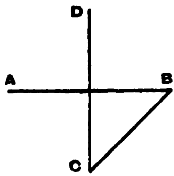


FIG. 27.

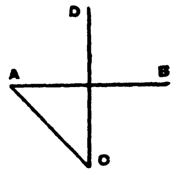


FIG. 28.

in the direction O, C. The resultant is B, C. (fig. 27). From here C, D begins to return, whilst B, A continues. The resultant is C, A. (Fig. 28).

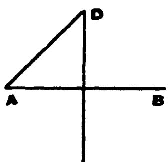


FIG. 29.

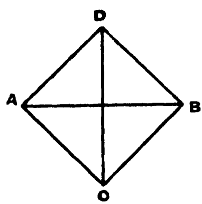


FIG. 30.

Similarly, the next resultant is A, D (Fig. 29); and the whole resultant figure is the square D, B, C, A. (Fig. 30). This is the other extreme phase. Between these two there may be any number of intermediate phases, the resultant figures being parallelograms more or less contracted; the midway phase like this (Fig. 31).

With a little mental effort these constant-motion figures may be translated into harmonic-motion

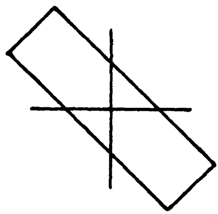


FIG. 31.

figures. The phases, amplitudes, and ratios are the same in both cases; the only difference is in the rates of motion in the course of the constituent vibrations. In constant motion the rate is unchanging ; in harmonic motion it is *always* changing from zero at the ends to zenith at the centre. So that in the case of the harmonic resultant corresponding to D, B., fig. 26, the D,O. constituent is at first acting at zero point, whilst A,B is acting at full swing. Consequently the harmonic

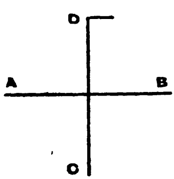


FIG. 32.

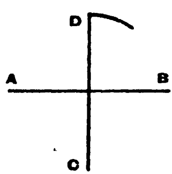


FIG. 33.

resultant starts off almost entirely in the direction of A,B. So (Fig. 32) ; but D,O is gaining velocity

whilst A,B is losing, and by the time their velocities have become equal the resultant has reached a point just half-way between D and B; it has in fact described *half a quadrant*, so (Fig. 33). From this point onwards D,O. gains ascendancy, reaching zenith at B, just as A, B. reaches zero. The quadrant D, B is thus completed (Fig. 34). In a similar manner the three succeeding quadrants are worked out, thus completing the whole circle (Fig. 35), D, B, C, A.

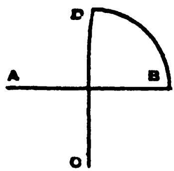


FIG. 34.

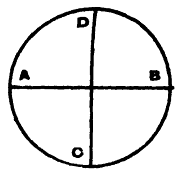


FIG. 35.

We thus find that the square in constant motion corresponds to the circle in harmonic motion; both being resultants of the combination of two to and fro motions acting at right angles in equal periods and with equal amplitudes.

For more complex combinations a numerical method of representing the vibratory motion will be found more convenient.

Observe that in the "graphic" method above described the figure is formed by connecting successive "resultants" of the combination of simultaneous portions of the vibration-paths. In the numerical method we find—not the actual

resultants of such combinations, but—their terminal points. The positions of these terminal points are determined by measurements—vertical and lateral—from the common centre of the combined vibrations.

Thus in the combination AB, CD,

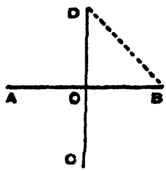


FIG. 36.

If the figure starts from point D the first thing to determine is, not the resultant *line* DB, but the resultant *point* B, and the line is obtained by simply joining the last point to the one before; the line of junction being straight for constant and curved for harmonic motion.

TO FIND THE POINTS.

Divide the vibration path into any convenient number of equal time-periods; for constant motion this will mean equal parts—not forgetting that one complete vibration-path goes twice over the vibration line; thus, if A O B is the vib. line the complete path will be from O to B, back to O, on to A, and back to O again. Now divide the path into 4 equal parts, using the radius O B as the *unit of measurement* and making

FIG. 37.

measurements right of the centre positive, left of the centre negative, use a heavy line above the *figure to distinguish negative movements.* Then, calling the starting-point O, the 1st division-point will be at B, 1 ; the 2nd at O, 0 ; 3rd at A, $\bar{1}$; 4th at O, 0; and the whole vibration will be represented by the series of figures 0 1 0 $\bar{1}$.

Point-numbers...	$\overset{4}{0}$	1	2	3
Point-distances...	0	1	0	$\bar{1}$

The 4th point being also the starting-point needs not to be separately written in. These figures serve equally well for the vertical vibration if we make measurements above the centre positive, below the centre negative. Thus, the square can be represented as follows :—

Point-numbers		$\overset{4}{0}$	1	2	3
Point-distances	Lateral vibration commencing at centre.	0	1	0	1
	Vertical vibration commencing at upper end.	1	0	$\bar{1}$	0

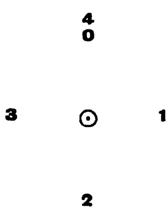


Fig. 38.

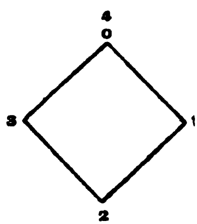


Fig. 39.

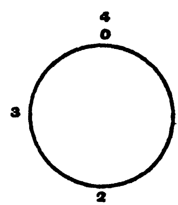


Fig. 40.

Measuring these distances from the centre of vibration O we get the points of the figure (Fig. 38).

These points connected by straight lines give the square (Fig. 39), connected by properly curved lines, they give the circle (Fig. 40).

But for harmonic figures it is better to take harmonic measurements—and plenty of them.

See fig. 19. To translate these measurements accurately into numerical form we must have recourse to trigonometry; but for many ordinary purposes it will be quite sufficient to take the series of figures 8, 7, 7, 6, 5, 4, 2, 1, as representing the harmonic division of a quarter-vibration (= half the vibration-line) into 8 equal time periods. For present purposes 4 will be sufficient. These we get by taking the above series in twos; thus: 15, 13, 9, 3. Now, if we divide this series by 10=1·5, 1·3, ·9, ·3, and then add the numbers together successively, we get a series 1·5, 2·8, 3·7, 4, which very conveniently indicates the comparative distances of *any* (harmonically) vibrating body from the vibration-centre at 4 successive equal time-periods.

Let this series represent the first quarter of a vibration; thus:
then the complete vibration will be shown thus:

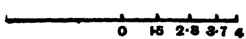


FIG. 41

Numerical representation of a Harmonic Vibration.

Point; distances	0	1·5	2·8	3·7	4	3·7	2·8	1·5	0	1.5	2·8	$\overline{3·7}$	$\overline{4}$	$\overline{3·7}$	$\overline{2·8}$	$\overline{1·5}$	0
Point numbers	0 16	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Principal phases	1st quarter				2nd quarter				3rd quarter				4th quarter				

To plot the circle, call this the lateral vibration; then the vertical vibration will be represented by the same series commencing at the end of the first quarter, and the circle will be thus represented:

Circle Points.

Vertical vibration	4	3·7	2·8	1·5	0	$\overline{1\cdot5}$	$\overline{2\cdot8}$	$\overline{3\cdot7}$	$\overline{4}$	$\overline{3\cdot7}$	$\overline{2\cdot8}$	$\overline{1\cdot5}$	0	1·5	2·8	3·7
Lateral vibration	0	1·5	2·8	3·7	4	3·7	2·8	1·5	0	$\overline{1\cdot5}$	$\overline{2\cdot8}$	$\overline{3\cdot7}$	$\overline{4}$	$\overline{3\cdot7}$	$\overline{2}\ \overline{8}$	$\overline{1\cdot5}$
Point numbers..	0 16	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Such figures are conveniently plotted on squared

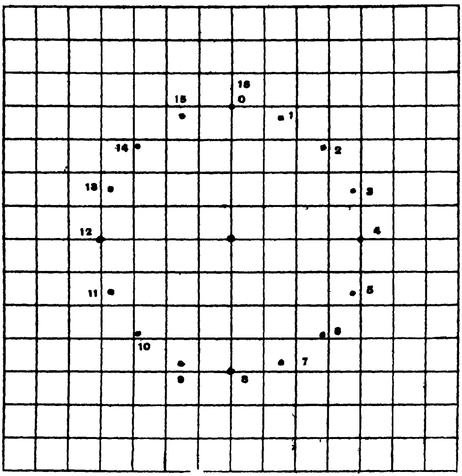


FIG. 42.

POINT DISTANCES OF CIRCLE MEASURED FROM THE CENTRE.

paper, taking the side of the square as the unit of measurement.

Thus we get the outline of points shown in Fig. 42.

There will be no difficulty in completing the circle by connecting the successive points.

$\frac{2}{1}$ Ratio.

Now, suppose that the vertical vibration moves twice as quickly as the other. Point 2 vertical will then coincide with point 1 lateral; point 4 vertical with point 2 lateral, etc., so that only the *alternate points* of the complete series representing the vertical vibration will coincide with the original series of the lateral vibration, thus,

Points for $\frac{2}{1}$ Ratio $\frac{1}{4}$ Phase.

Vertical vibration	4	2·8	0	$\overline{2\cdot8}$	$\overline{4}$	$\overline{2\cdot8}$	0	2·8	4	2·8	0	$\overline{2\cdot8}$	$\overline{4}$	$\overline{2\cdot8}$	0	2·8
Lateral vibration	0	1·5	2·8	3·7	4	3·7	2·8	1·5	0	$\overline{1\cdot5}$	$\overline{2\cdot8}$	$\overline{3\cdot7}$	$\overline{4}$	$\overline{3\cdot7}$	$\overline{2\cdot8}$	$\overline{1\cdot5}$
Point numbers—	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

The curve is shown in Fig. 43.

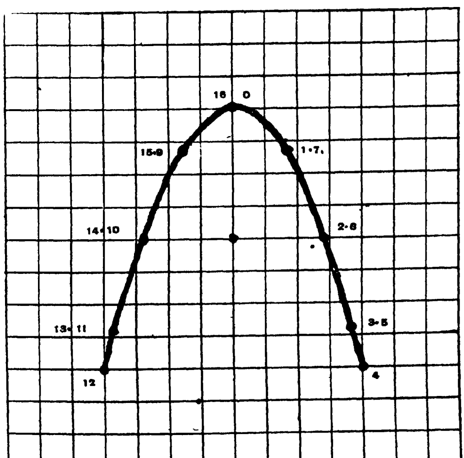


FIG. 43.

$\frac{2}{1}$ RATIO : $\frac{1}{4}$ PHASE MEASURED FROM THE CENTRE.

This is what is known as a "retrogressive" curve; it will be observed that the vibration is not com- *plete until the moving point has been twice over* the outline. In this phase the figure is a parabola.

The initial phase of this combination shows a very different figure. For this we use the same numbers, but both vibrations start simultaneously from the centre.

Points for $\frac{2}{1}$ Ratio initial phase.

Vertical vibration	0	2.8	4	2·8	0	$\overline{2{\cdot}8}$	$\overline{4}$	$\overline{2{\cdot}8}$	0	2·8	4	2·8	0	$\overline{2{\cdot}8}$	$\overline{4}$	$\overline{2.8}$
Lateral vibration	0	1·5	2·8	3·7	4	3·7	2·8	1·5	0	$\overline{1{\cdot}5}$	$\overline{2{\cdot}8}$	$\overline{3{\cdot}7}$	$\overline{4}$	$\overline{3{\cdot}7}$	$\overline{2{\cdot}8}$	$\overline{1{\cdot}5}$
Point numbers ..	0 16	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

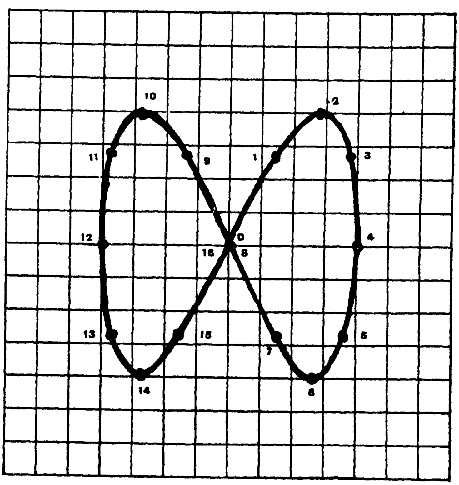


FIG. 44.

$\frac{2}{1}$ RATIO: INITIAL PHASE MEASURED FROM THE CENTRE.

So far we have dealt only with pairs of single

vibrations. These may be added, subtracted, multiplied, and modified indefinitely by performing the necessary operations on their point-numbers and plotting out the results.

NOTATION.

The following simple form of notation will serve to make such operations intelligible.

Let any complete vibration be represented by a short horizontal line, so———. The rate of vibration is shown by a figure above or below the line, vertical, above; lateral, below. The phase is indicated by the position of the figure on the line. Thus, $\underset{1}{——}——^{2}$ represents the curve shown in fig. 43; lateral vibration commencing at the initial phase, vertical vibration—twice as quick—at quarter phase. So again $——\overset{2}{\underset{1}{—}}——$ represents fig. 44, both vibrations commencing at the initial phase. So too $——\underset{1}{—}——^{1}$ represents the circle, fig. 42, both vibrations having the same rate; lateral—initial phase; vertical—quarter phase. The half-phase (seldom used) may be shown thus $——\overset{<1}{—}——$, and ¾ phase thus $^{1}——\ ——$

Now suppose we wish to combine $—\underset{1}{—}—^{2}$ and $——\overset{2}{\underset{1}{—}}——$, figs. 43 and 44. They *may* be combined exactly as they stand $——\overset{2}{\underset{\underset{1}{1}}{—}}——^{2}$, and the result is not un-interesting. But we shall get more characteristic results by taking one of the lateral vibrations as a vertical—and one of the verticals as a lateral vibration; thus $——\underset{\underset{1}{2}}{—}——^{\overset{2}{1}}$ This, it will be seen,

1

is simply adding together the two circles $\underset{1}{\text{————}}{}^{1}$ and $\underset{2}{\text{————}}{}^{2}$ one of which goes twice as quickly as the other. In this case the two circles are moving in the same direction—they are "concurrent," if we take one of them at three-quarter phase $({}^{1}\cdot\underset{1}{\text{————}})$ they will move in opposite directions—"counter-current," and the combination will be ${}^{1}\underset{\substack{2\\1}}{\text{——}}\quad{}^{2}$

Let us now work out the first of these combinations $\underset{\substack{2\\1}}{\text{————}}{}^{\substack{2\\1}}$. For the compound vertical vibration add together the two series of numbers used for fig. 43, taking care to start *both* series at quarter-phase.

For the compound lateral vibration add together, just as they stand, the numbers used for fig. 44. We thus get the compound series of point-distances as below.

Point-distances of $\underset{\substack{2\\1}}{\text{————}}{}^{\substack{2\\1}}$

Vertical	8	6·5	2·8	$\overline{1·3}$	$\overline{4}$	$\overline{4·3}$	$\overline{2·8}$	$\overline{·9}$	0	·9	$\overline{2·8}$	$\overline{4·3}$	$\overline{4}$	$\overline{1\ 3}$	2·8	6.5
Lateral	0	4·3	6·8	6·5	4	·9	$\overline{1·2}$	$\overline{1.3}$	0	1·3	1 2	$\overline{·9}$	$\overline{4}$	$\overline{6·5}$	$\overline{6·8}$	$\overline{4·3}$
Point Numbers..	0 16	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

From these measurements the figure is easily plotted.

The centre of vibration is at point 8.

This curve is known to mathematicians as the "Trisectrix." The length of a straight line from

the starting point to centre of vibration is exactly twice as much as that from the intersection-point at the bottom of the loop to the same centre.

This becomes evident from the figures, as well

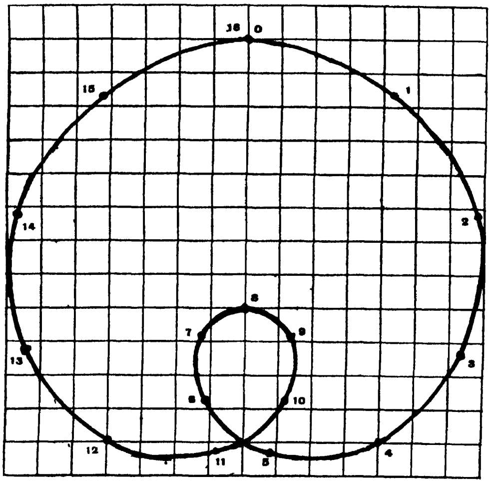


FIG. 45. TRISECTRIX.

as from the diagram, when the vibration-path is divided into 12 or 24 periods.

In music, this curve represents the octave—because its vibration-ratio is the same 2 : 1 : it is also typically representative of its class, the special features of the ratio being developed to the utmost limit of individuality. This is due to two principal facts: 1st, each of the four vibrations has the same

amplitude; 2nd, the vertical and lateral vibrations are in extreme opposition, one set commencing at the centre, the other at the end of the path—in other words this is the most definite "phase" of the combination. In all cases there is an unlimited variety of figures for every ratio, because the form depends just as much upon the *amplitude*-ratio as it does upon the *period*-ratio; whereas the unqualified term "ratio" is generally understood to refer only to the latter.

This dependence of the figure-form on the amplitude ratio is sometimes annoyingly illustrated in the working of a badly mounted pendulum, when (in consequence of the amplitudes of the various vibrations diminishing at different rates) one vibration survives the others and the figure ends in a straight line. Although vibrations of equal amplitude give rise to figures of the most marked individuality, it is probable that the most characteristic figures are those in which the amplitude-ratio and the period-ratio are exactly the opposite of each other, the amplitudes of the vibrations being inversely as the vibration-numbers. At least, this is the order of amplitude naturally pertaining to harmonic combinations.

To work out the combination in fig. 45 with this amplitude-ratio, take the figures in precisely the same order as in Fig. 45, but taking care to divide the 2-vibration series by 2 before adding together

for both compound vibrations. This will give the following series:

$$\text{Point distances of } \frac{\overset{2\,(\div 2)^{*}}{1}}{\underset{1}{2\,(\div 2)}}$$

Point Numbers	0 (16)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Vertical......	6	5·1	2·8	·1	$\overline{2}$	$\overline{2{\cdot}9}$	$\overline{2{\cdot}8}$	$\overline{2{\cdot}3}$	$\overline{2}$	$\overline{2{\cdot}3}$	$\overline{2{\cdot}8}$	$\overline{2{\cdot}9}$	$\overline{2}$	·1	2·8	5·1
Lateral......	0	2 9	4·8	5·1	4	2·3	·8	·1	0	$\overline{{\cdot}1}$	$\overline{{\cdot}8}$	$\overline{2{\cdot}3}$	$\overline{4}$	$\overline{5{\cdot}1}$	$\overline{4{\cdot}8}$	$\overline{2{\cdot}9}$

Plotted out, this gives fig. 46, known to mathematicians as the "Cardioid."

Amongst the unlimited series of T. E. figures there are two special varieties that stand pre eminent in their capacities for beauty and diversity of form; these are the 3 : 1 ratio counter-current and 5 : 1 ratio con-current. This peculiarity depends primarily on the openness of structure specially appertaining to these figures under a suitable balance of conditions; and this, again, depends on the angle of deflection proper to the ratio. A brief examination of the figures will reveal the fact that with the counter-current series the angle of deflection diminishes precisely in proportion to the magnitude of the vibration-ratio, ending with zero (0°) when the length of the deflecting pendulum becomes infinite. Conversely, the deflecting angle increases in the opposite direction

*In the phase and ratio notation observe that operations on the amplitude are signified by small figures in brackets near to the numbers representing the affected vibrations. Thus 2 (÷2) means that the vertical vibration in the deflecting circle (or ellipse) is taken at half the amplitude of the fundamental vibration.

(with the increasing magnitude of the vibration-ratio), ending with 180° when the deflector becomes infinitely short. In both cases the limiting "figure" is indistinguishable from a straight line. At the central position between these two infinite series of gradations, the angle of

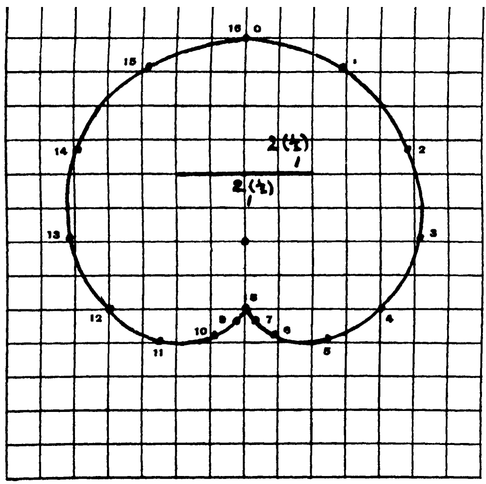


FIG. 46.

deflection is 90° and the "figure" is approximately a square—more or less rounded at the corners according to the amplitude ratios—1 : 3 gives the maximum squareness. The central square in the counter-current series is the 3 : 1 figure; with concurrent figures the central square is found at the vib.-ratio of 5 : 1, amplitude-ratio 1 : 5.

It has already been shown (page 100) that 2, 3 and 5 are the foundation numbers of all the rhythm-combinations proper to written music; and above considerations seem to show that their functions are not less significant in relation to physical operations generally, and to the constitution of T. E. figures particularly.

The phenomena of audible music and of twin-elliptic vibrations are, of course, not strictly analogous throughout; but in both cases 5 and 3, that is $\frac{5}{1}$ and $\frac{3}{1}$, are the basic ratios of the most highly characteristic rhythms possible to the two systems. This explains to some extent why $\frac{3}{1}$ is the most important of T. E. figures. $\frac{5}{1}$ is not readily attainable under ordinary conditions, because of the great height required for the suspension-hook.

As a further illustration of the numerical method of delineation let us take the 3 : 1 figure in its most characteristic form, that is, with counter-current phase and amplitude-ratio = 1 : 3, the quicker vibration having one-third the amplitude of the slower.

The arrangement of vibrations will be $\dfrac{3\ (\div 3)\qquad\qquad 1}{\substack{3\ (\div 3)\\ 1}}$

and the point distances as follows :—

Point-distances of $\dfrac{3\ (\div 3)\qquad\qquad 1}{\substack{3\ (\div 3)\\ 1}}$

Vertical	2·7	3·2	3·7	2·7	0	$\overline{2\cdot7}$	$\overline{3\cdot7}$	$\overline{3\cdot2}$	$\overline{2\ 7}$	$\overline{3\ 2}$	$\overline{3\cdot7}$	$\overline{2\cdot7}$	0	2·7	3·7	3·2
Lateral	0	2·7	3·7	3·2	2·7	3·2	3·7	2 7	0	$\overline{2\cdot7}$	$\overline{3\cdot7}$	$\overline{3\cdot2}$	$\overline{2\cdot7}$	$\overline{3\cdot2}$	$\overline{3\cdot7}$	$\overline{2\cdot7}$
Point Numbers	0 16	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

With these data the figure is easily plotted as under.

Of course the student will remember that just as in 2 : 1 ratio the point-distances for the quicker vibration were found by taking every second of the original series, so with 3 : 1 we take every third of

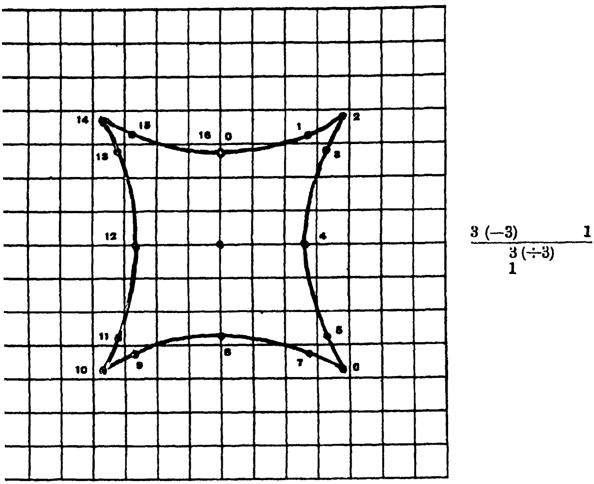
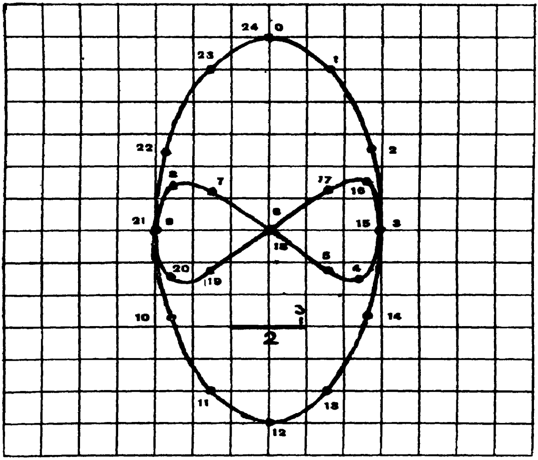


FIG. 47.

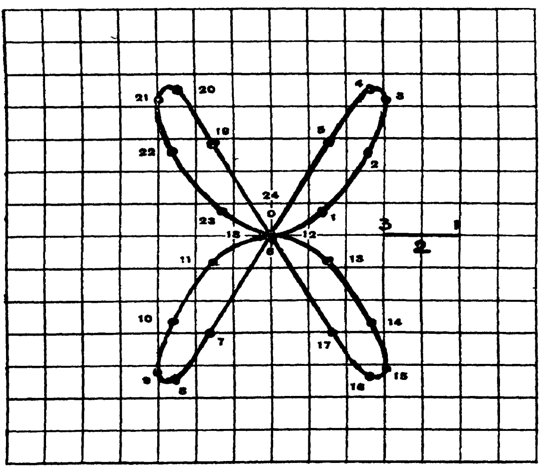
the same to make the series for the quicker vibration.

The two following figures (with which I shall conclude this portion of the subject) contain neither circular nor elliptic motion ; they are combinations of three single vibrations only, and cannot be produced by wheel-work, except by some very



24
0
23
1
22
2
8
7
17
16
21
9
6
15
3
18
5
20
19
4
10
14
3
1
2
11
13
12

FIG. 48.



21
20
4
3
19
5
22
2
24
0
23
1
18
12
3
1
2
6
11
13
10
14
7
17
9
8
16
15

FIG. 49.

cumbersome arrangement in which each straight line motion is itself the product of two circular motions.

Three-straight-line figures ;

Point-distances of $\frac{\overline{\quad\quad\quad}^{3}_{1}}{2}$

Vertical*
6 5 2 6 0 1·5 1·3 0 1·3 1·5 0 2·6 5 6 5 2·6 0 1·5 1·3 0 1·3 1·5 0 2·6 5
Lateral
0 1·5 2·6 3 2·6 1·5 0 1·5 2·6 3 2 6 1·5 0 1·5 2·6 3 2·6 1 5 0 1·5 2·6 3 2 6 1·5
Point Numbers
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
24

Point-distances of $^{3}\overline{\quad\quad\quad}^{1}_{2}$

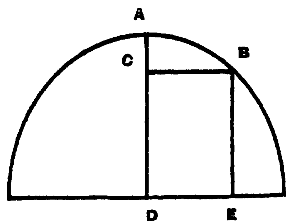


FIG. 50.

Vertical
0 ·8 2·6 4·2 4·5 2·9 0 2·9 4·5 4·2 2·6 ·8 0 ·8 2·6 4·2 4·5 2·9 0 2·9 4·5 4·2 2·6 ·8
Lateral
0 1.5 2·6 3 2·6 1·5 0 1·5 2·6 3 2 6 1·5 0 1·5 2·6 3 2·6 1·5 0 1·5 2·6 3 2·6 1·5
Point Numbers
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
24

Anyone wishing to pursue this system of curve-delineation may easily do so with a little

*It should be observed that we have here divided the single vibration into 24 equal time-periods as follows.—

0 ·8 1·5 2·1 2·6 2·9 3 2·9 2·6 2·1 1·5 ·8 0 ·8 1·5 2·1 2·6 2·9 3 2·9 2·6 2·1 1·5 ·8
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
24

This is merely a matter of convenience ; it is simply necessary that the values given to successive point-distances should be proportional to the sines of the arcs to which they correspond. A B, arc. C B, sine. D E, point-distance.

application ; no mathematical knowledge is necessary beyond what is open to every intelligent observer, and the subject is full of interest and fascination.

I have myself plotted some hundreds of figures embodying combinations of vibrations which have never yet been attempted by any mechanical contrivance, but still there is room for any number of industrious explorers. What is required to enable any such figures to be worked out mechanically is nothing more than a mechanism for combining any number of harmonic motions in one straight line. Two such combinations working at right angles to each other could produce the resultants of any possible grouping of vibrations. The truth of this statement is demonstrated by the working of the "Telautograph."

(IV.)—VIBRATION-FIGURES WHOLLY ARTIFICIAL.

Division II.—Mechanical.

SO far as I am aware these are produced entirely by means of wheel-work. A great variety of machines have at times been made for this class of work, in most cases apparently with the intention of combining two or more circular motions, and without any reference to the fact that a circle is the resultant of two harmonic vibrations

of equal periods and equal amplitudes acting at right angles to each other.

It is only of late years that the term harmonograph has been applied to this class of apparatus, the earlier specimens being designed chiefly for purposes of ornamental turning. The most effective machine of the kind that has come before the public is that which is known as "The Geometric Chuck" The history and capabilities of this important mechanism (from the ornamental turners' point of view) has been most ably described by Sir Thos. Bazley in his "Index to the Geometric Chuck" (published by Holtzapffel and Co., 1875). The work is illustrated with 3,500 specimen figures, all produced by means of the "Chuck" *and its accessories.*

Every figure is the resultant of two or more circular motions acting simultaneously on a single point. The forms selected are mostly typical phases of the most important combinations, and it is just here that such machines may have an important scientific value, as by their means any phase whatever can be selected and written off with equal facility and precision.

Some of the figures involve as many as 5 circular motions—equal to 10 vibrations, but it is probable that in many cases these complex motions are mutually destructive. The whole book requires interpreting into vibration equivalents. Many of the figures are awfully wonderful,

and the work is altogether the most important treasury of outline vibration-forms extant—although the word "vibration" is not mentioned throughout!

A word of caution is necessary for those who may consult the book for information about vibration-forms. Mr. Bazley's classification is adapted to the exigencies of the Geometric Chuck, and for this reason Concurrent and Counter-current figures of the same vibration-ratio will not be found associated together, but, instead, figures having the same number of loops, whether internal or external. These associated figures have no natural relationship to each other; their ratios are always different, and sometimes widely different, but the "companion" figures frequently referred to are figures of the *same ratio with special phase-differences.*

At present the majority of Mr. Bazley's figures are geometrical enigmas. The very few persons who possess apparatus of the same description as Mr. Bazley's should be able to reproduce his figures; but, even so, it will not in all cases be an easy matter to find their vibration equivalents.

Here it may be well to observe that the first 200 figures are "twin-elliptics"; they all come within the limits of the twin-elliptic pendulum. So, again, are figs. 226 to 251, 428 to 433, and 833 to 850; these all belong to the 3 : 1 ratio; figs. 806 to 819 and 1258 to 1267 are twin-elliptics of the 2 : 1 ratio;

whilst figs. 460 to 471 and 252 to 263 are tri-une, being compounded of ellipses in the ratio 1 : 3 : 5; and *finally* figs. 220 to 225 are tri-une elliptics in the ratio 1 : 2 : 3. Nearly all the rest of the figures are much more complex, and most of those towards the end of the book are composite—consisting of several distinct figures in one complex diagram. Withal, the whole assortment is but a small fraction of the vast variety of vibration-figures yet to be made known,

I have said that Bazley's "Index" is the most important work on Vibration-figures that has hitherto come before the public, and the apparatus therein described may well be classed in the same category.

There is now in existence, however, and in good working order, an apparatus—not yet perfect, but far in advance of the ancient "Chuck" and its congeners. This incomparable piece of mechanism is the property of J. E. Austin, Esq., of West Court, Detling, near Maidstone, and is the product of the joint work of the most experienced veterans in practical kinematics now living. The chief distinctive feature in the apparatus is the mechanism providing for the production of harmonic motion in a straight line, and for its gradual increase or decrease in amplitude. By means of this unique contrivance the harmonic components of circular (or elliptic) motion can be dealt with separately, so that variations in their relative intensities can be

graphically and definitely represented at will. Let it be observed here that the intensity of a vibration is its amplitude in a sense-aspect.

For the purpose of providing illustrations of the interaction of simple vibratory motions, therefore, it will be seen that Mr. Austin's apparatus has important advantages; any one of the constituent vibrations of a figure can be made to increase whilst the others are decreasing in amplitude, and the rate of such increase or decrease can be regulated at will.

MR. AUSTIN'S FIGURES.

The few figures I have selected as specimens of Mr. Austin's work are of the simplest order. They are curves of two vibrations only (pages 110 and 111), commonly known as Lissajous figures.

The upper one on Plate XXV. represents the unison (1 : 1) or $\frac{1}{1}$ when the two vibrations, acting at right angles, arrive at the centre coincidently. With equal amplitudes the result is a straight line diagonal to the right angle. But here is shown the effect when one of the vibrations—say, the vertical—begins with its full amplitude and decreases to zero, whilst the lateral begins at zero and increases to an equal amplitude with the vertical. This gives a series of straight lines at all possible angles, proportional to the united amplitudes. The whole effect might be noted thus $\frac{1 > 0}{0 < 1}$

In the lower one, Plate XXV. (Unison $= \frac{1 > 0}{0 < 1}$) we have the same interval with the same amplitudes,

increasing and decreasing as before, but at opposite phases. With constant amplitudes the result would be the circle as Fig. 42. But the constantly *changing* amplitudes give rise to a succession of ellipses beginning with the vertical straight line, going through all possible degrees of widening and narrowing, and ending with the lateral straight line.

On Plate XXVI. in the upper figure (Octave= ———— $\frac{2<0}{0>1}$) we have a group of parabolas representing all possible sections of the cone parallel to one side.

The figure with constant amplitudes is the same as that shown on page 111.

The lower figure on plate XXVI. (perfect fifth ———— $\frac{2>0}{0<3}$) with constant amplitudes is one of the commonest figures obtainable with the ordinary double pendulum.

The outline corresponding to equal amplitudes —like this () —can be easily traced in the present complex figure, commencing at the 7th point from the top.

The figure on plate XXVII. (perfect fourth ———— $\frac{4>0}{0<3}$) like all the others begins with a vertical and ends with a horizontal straight line. The intermediate figure with equal amplitudes commencing at the eighth point from the top is the common "overhanded knot," and sometimes called the "true lover's knot."

These very interesting figures are, of course, quite

out of the reach of any pendulum arrangement; but figures of a similar class, giving the effect of one sustained vibration, can be produced by putting a drag on one of the weights of a double pendulum.

There is now an open opportunity for some enthusiastic student, with money and energy at his disposal, to do for twin-elliptics what Mr. Austin has done for Lissajous' figures.

(V.)—VIBRATION-FIGURES, PARTLY NATURAL AND PARTLY ARTIFICIAL.

THE forms of apparatus that have been devised for the production of figures coming within this category are very numerous. Amongst the chief workers in this field of labour may be mentioned Wheatstone, Melde, Lissajous, Blackburn, Airy, and Tisley. My own work, consisting chiefly in the production and development of the T. E. P., would, of course, have been impossible without the preceding efforts of these and other workers.

These natural-artificial methods of producing vibration figures depend for their excellence chiefly on the suppression of the artificial element in the work—reducing all interference with Nature's free hand to the lowest limit attainable. This is what I think has been achieved in the twin-elliptic pendulum, of which I will first give a short description.

Essentially the T. E. P. consists of a heavy pendulum swung from a single point and carrying a second pendulum (called the "deflector") swinging from a point in the main pendulum as near as may be to its centre of gravity. The actual weights may

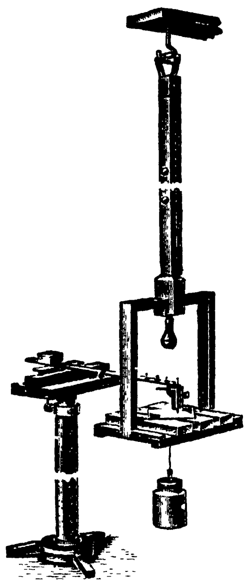


FIG. 51.

vary almost without limit; generally the main pendulum weights vary from about 16lbs. to 60lbs.; the deflector from 4 to 12lbs.

The annexed diagram should be largely self-explanatory, but a few observatious on its principal features may prove helpful.

It will be found on inspection that—beyond the pen-point and the atmosphere—the only point of friction for the main pendulum is between the ball and the polished cavity of the steel slab; these are both well-hardened, so that friction is reduced to the lowest minimum consistent with durability. Friction at the pen-point is easily regulated by loading or unloading the pen-holder with fine-adjustment weights, and some interesting effects in the development of figures are often attainable by this means.

Direct minimization of friction means indirect minimization of strain, and it is this which ensures the possibility of obtaining more perfect results with the T. E. P. than with any other form of vibration apparatus.

BALANCE.

The very best work is obtained when friction at the pen-point is reduced to a minimum, but then there must be a suitable balance of all the other conditions. If, for example, the pen-friction should be small, whilst the weights are heavy and the pen coarse, the result will be that the lines of the figure will be too close together, and may actually form one continuous patch of ink. This again is much more likely to happen with a figure of simple ratio and short outline than with one of complex ratio and long outline. But if the pen should be sufficiently fine or the outline sufficiently long, the

lines will remain separate, and some new and beautiful example of the supremacy of order will be the result. Another set of conditions with a fine pen will give an uninteresting patch of network, whilst the same conditions with a coarse pen will result in a gorgeous series of intersection figures. So it is that any arrangement of the pendulum will yield beautiful results if the various conditions are well-balanced.

The most important features in the apparatus are the points of suspension. As already explained, these are devised to ensure the least possible amount of friction; but it is also very desirable that the main suspension point should have the utmost firmness attainable. To this end the hook is made strong enough to carry at least ten times the maximum load required of it; but it is also requisite that the resisting forces by which the hook is supported should be either excessively strong or free from inequalities.

The reason for this desideratum is to be found in the fact that any inequality in the forces of resistance must give rise to a similar inequality in the loss of active energy by the moving pendulum in the same direction. This means that one of the vibrations of the primary ellipse will die out at an accelerated rate and, consequently, the figure will tend to degenerate into a straight line—at right angles to the direction of infirmity.

Symmetry, so far as convenient, is desirable

throughout the distribution of the pendulum mass; but the chief momentum weight admits of considerable variety in form and material. It is most convenient when constructed in sections.

The deflector is provided with a fine adjustment screw having sixteen threads to the inch. By this means the actual length of the deflecting pendulum is easily regulated within $\frac{1}{100}$ of an inch or less.

ACTION OF THE T.E.P.

Being suspended from a single point, the whole system is free to move—that is, to vibrate—in any direction whatever in a practically horizontal plane: its rate of movement is inversely as the square of its length. Considering the main pendulum without the deflector, any impulse or any number of impulses that may be given to its mass are always equivalent to two impulses acting at right angles to each other, and their resultant effect must always be some form of ellipse between the circle and the straight line. The same observation is equally true of the deflector considered by itself. But in the course of actual operation the main pendulum without the deflector has no distinct existence, and the real main pendulum is the whole system, including the deflector. And although it is true that neither the ellipse proper to the whole system, nor that which is proper to the deflector, can be obtained separately, whilst the two are in a condition to work conjunctively, yet these two

ellipses are the actual components of the figures produced by the ordinary working of the apparatus as a whole. Endless as is the variety of figures obtainable from the action of the T.E.P., they consist, in every case, of two such elliptic motions working together at various rates and amplitudes. Some people imagine that these figures are not inexhaustible; this arises from lack of devout attention to the subject. Bad work is all alike incapable of classification ; it has no individuality, and, therefore, no variety, and is distinguished only by negligence. Good work is distinguished by intelligent attention ; its character and individuality are strong and striking in proportion to the truth-loving devotion with which it is carried out; its development and variety are co-extensive.

Both the main pendulum and the deflector admit of practically indefinite variations in length, and, so, of corresponding variations in the ratios (or comparative rates) of their vibrations, the rate being inversely as the square of the length. There is, therefore, available for pendulum work an unlimited variety of ratios, each having its own proper series of figures with varying phases, amplitudes and rates of decrease. The most important and most interesting ratios are, however, the simplest; and these are they that correspond to the principal musical intervals. Nevertheless, the parallelism between figures and intervals is not readily evident, because the system of vibration-figures has no

limit! Whereas the system of musical intervals, as already shown (page 100), is a selection from the vibration-system, limited to the powers of 2, 3, and 5.

And twin-elliptic figures, on the other hand, although consisting of two doubly infinite series above and below 3 : 1, con-current and counter-current, are yet but a small portion of the whole field of vibration-figures generally.

Some of the most important intervals, for example, are those ranging from unison (1 : 1) to the "perfect fifth" (3 : 2), and these are practically outside the limits of the T.E.P., whereas the most important T.E.P. figure is undoubtedly that which corresponds to the interval of 3 : 1.* (See page 118 for further information on this subject.)

Annexed is a list of the principal ratios available for the T.E.P. :—

Diagram of the principal ratios between 3 : 2 and 6 : 1 in order of magnitude, showing their approximate positions in a geometrical progression by comparison with the successive powers of $\sqrt[100]{4} \times (1{\cdot}5)$:

Powers of $\sqrt[100]{4} \times 1{\cdot}5$			Ratios.		
Index.	Product.		Decimal.	Fractional.	
0	1·5	..	1·500	3 : 2	Perfect Fifth
1	1·521				
2	1·542	..	1·555	14 : 9	
3	1·564	..	1·571	11 : 7	
4	1·585				
5	1·607	..	1·600	8 : 5	Minor Sixth
6	1·630	..	1·625	13 : 8	

*This, although forming no part of the authentic scale is, as shown elsewhere, the centre of its harmonic basis.

Powers of $\sqrt[100]{4 \times 15}$			Ratios.		
Index.	Product.		Decimal.	Fractional.	
7	1·653	..	1·666	5 : 3	Major Sixth
8	1·676	..			
9	1·699	..	1·700	17 : 10	
10	1·723	..	1·714	12 : 7	
11	1·747	..	1·750	7 : 4	Harmonic Seventh
12	1·771	..	1·777	16 : 9	Dominant Seventh
13	1·796	..	1·800	9 : 5	Tonic Seventh
14	1·821	.	1·833	11 : 6	
15	1·846	..	1·857	13 : 7	
16	1·872	..	1·875	15 : 8	Major Seventh
17	1·898	..	1·900	19 : 10	
18	1·925				
19	1·952				
20	1·979				
21	2·006	..	2·000	2 : 1	Octave
22	2·035				
23	2·063				
24	2·092	..	2·100	21 : 10	
25	2·121	..	2·111	19 : 9	
			2·125	17 : 8	
26	2·151	..	2·142	15 : 7	
			2·166	13 : 6	
27	2·181				
			2·200	11 : 5	
28	2·211				
			2·222	20 : 9	
29	2·242				
			2·250	9 : 4	Major Ninth
30	2·274				
			2·285	16 : 7	
31	2·305	..	2·300	23 : 10	
			2·333	7 : 3	Harmonic Minor Tenth
32	2·337				
33	2·370				
			2·375	19 : 8	
			2·400	12 : 5	Minor Tenth
34	2·403				

Powers of $\sqrt[100]{4 \times 1\cdot5}$ Index.	Product.		Ratios. Decimal	Fractional.	
			2·428	17 : 7	
35	2·437				
			2·444	22 : 9	
36	2·470				
			2·500	5 : 2	Major Tenth
37	2·505				
38	2·540				
			2·555	23 : 9	
			2·571	18 : 7	
39	2·576				
			2·600	13 : 5	
40	2·612				
			2·625	21 : 8	
41	2·648				
			2·666	8 : 3	Perfect Eleventh
42	2·685				
			2·700	27 : 10	
43	2·722	..	27·14	19 : 7	
			2·750	11 : 4	Harmonic Eleventh
44	27.60				
			2·777	25 : 9	
45	2·798				
		.	2·800	14 : 5	
46	2·837	..	2:833	17 6	
			2·857	20 : 7	
47	2·876	..	2·875	23 . 8	
			2·888	26 : 9	
48	2·917	..	2·900	29 : 10	
49	2·959				
50	3·000	..	3·000	3 : 1	Perfect Twelfth
51	3·042	..			
52	3·084				
			3·111	28 9	
53	3·127	..	3·125	25 : 8	Augmented Twelfth
			3·143	22 · 7	
54	3·170	..	3·166	19 : 6	
			3·200	16 : 5	Minor Thirteenth
55	3·215				
			3·222	29 : 9	

Powers of $\sqrt[100]{4 \times 1\cdot5}$ Index.	Product.		Ratios. Decimal.	Fractional.	
56	3·260	..	3·250	13 : 4	Harmonic Thirteenth
			3·285	23 : 7	
57	3·305	..	3·300	33 : 10	
			3·333	10 : 3	Major Thirteenth
58	3·352				
			3·375	27 : 8	
59	3,398	..	3·400	17 : 5	
			3·428	24 : 7	
60	3·446	..	3·444	31 : 9	
61	3·494				
			3·500	7 : 2	Harmonic Fourteenth
62	3·542				
			3 555	32 : 9	Dominant Fourteenth
63	3·592	..	3·571	25 : 7	
			3·600	18 : 5	Tonic Fourteenth
			3·625	29 : 8	
64	3·642				
			3 666	11 : 3	
65	3·693				
			3·700	37 : 10	
66	3·744	..	3·714	26 : 7	
			3·750	15 : 4	Major Fourteenth
67	3·796	..	3·777	34 : 9	
			3·800	19 : 5	
68	3·849	..	3·833	23 : 6	
			3·857	27 : 7	
			3·875	31 : 8	
69	3·902	..	3 888	35 : 9	
70	3·959	..	3·900	39 : 10	
71	4·014	..	4·000	4 : 1	Double Octave
72	4·070				
			4·142	29 : 7	
73	4·127				
			4·166	25 : 6	
74	4·184				
			4·200	21 : 5	
75	4·242				
			4·250	17 : 4	
76	4·302	..	4·285	30 : 7	
77	4·362	..	4·333	13 : 3	
			4·400	22 5	

Powers of $\sqrt[100]{4 \times 1\cdot5}$			Ratios.	
Index.	Product.		Decimal.	Fractional.
78	4·423			
			4·428	31 · 7
79	4·484			
			4·500	9 : 2
80	4·547			
			4·571	32 : 7
81	4·611	..	4·600	23 : 5
82	4·675	..	4·666	14 : 3
83	4·740	..	4·714	33 : 7
			4·750	19 . 4
84	4·806	..	4·850	24 . 5
			4·833	29 : 6
85	4·873	..	4·857	34 : 7
86	4·941			
			5·000	5 : 1
87	5·009			
88	5·080			
			5·142	36 : 7
89	5·151			
			5·166	31 : 6
90	5·223	..	5·200	26 · 5
			5·250	21 : 4
91	5·295	..	5·285	37 : 7
			5·333	16 : 3
92	5·370			
			5·400	27 : 5
93	5·445	..	5·428	38 : 7
			5·500	11 : 2
94	5·523			
			5·571	39 : 7
95	5·594			
			5·600	28 : 5
96	5·677	..	5·666	17 : 3
			5·714	40 : 7
97	5·756	..	5·750	23 : 4
			5·800	29 : 5
98	5·834	..	5·833	35 : 6
			5·857	41 : 7
99	5·917			
100	6·000	..	6·000	6 : 1

This diagram is designed to facilitate the estimation of distances between one ratio and another.

The unit of measurement is the index-number.

Example.—The distance between 3 : 1 and 5 : 2 is 50—37=13 units.

Between 5 : 2 and 7 : 3 is 37—32=5 units, etc.

For every possible ratio there are two principal classes of figures, "Concurrent," in which the two elliptical movements are in the same direction, and "Counter-current," in which they are in opposite directions. These again can be very largely varied, according to the variation of one or both of the ellipses between the circle and the straight line.

Generally speaking, the best figures are those in which both ellipses are as nearly as possible circular. Precise phases and amplitudes, whether typical or otherwise, cannot be assured by mere hand-work; in this respect mechanical contrivances like the Geometric Chuck have decided advantages, enabling any phase or amplitude to be accurately represented; this is their chief use and merit.

Comparing pendulum figures with mechanical figures, it is easy to get the impression that the latter involve more accurate movements than the former, but this is quite erroneous. The pendulum expresses facts to which the mechanism is dead. The chief characteristic of mechanical figures is deadly rigidity, but that of pendulum figures is vital fluency; whilst the incomparable accuracy of pendulum motion is unequalled by that of any mechanical contrivance. The fixed mechanism

incorporates only what has practically been predetermined by the operator; the fluent pendulum gives ever new results with surprising details beyond the reach of imagination to conceive.

Following are a few outlines of important twin-elliptic figures from Bazley's "Index." Only the chief phases (Concurrent and Counter-current) are shown, and only two amplitude-ratios. In "Equal" amplitudes all four vibrations are equal. "Inverted" amplitudes means that the ratios of the amplitudes of the two ellipses are inversely as the vibration-ratios: thus in 3 : 1 the amplitudes would be 1 : 3.

Typical outlines of T.E. figures corresponding to principal ratios, taken from "Bazley's Index," in order of pitch:

No.	Ratio.	Amplitudes.	Phase.	Bazley's Number.
1 ..	3 : 2 ..	Equal ..	Concurrent. ..	197
2 ..	3 : 2 ..	Inverted ..	Concurrent ..	196
3 ..	3 : 2 ..	Inverted ..	Counter-current ..	200
4 ..	2 : 1 ..	Equal ..	Concurrent ..	4
5 ..	2 : 1 ..	Equal ..	Counter-current ..	18
6 ..	2 : 1 ..	Inverted ..	Counter-current ..	16
7 ..	2 : 1 ..	Inverted ..	Concurrent ..	2
8 ..	7 : 3 ..	Inverted ..	Counter-current ..	
9 ..	5 : 2 ..	Equal ..	Counter-current ..	47
10 ..	5 : 2 ..	Inverted ..	Counter-current ..	45
11 ..	3 : 1 ..	Equal ..	Counter-current ..	28
12 ..	3 : 1 ..	Equal ..	Concurrent ..	8
13 ..	3 : 1 ..	Inverted ..	Concurrent ..	6
14 ..	3 : 1 ..	Inverted ..	Counter-current ..	26
15 ..	4 : 1 ..	Equal ..	Concurrent ..	13
16 ..	4 : 1 ..	Inverted ..	Concurrent ..	11
17 ..	5 : 1 ..	Equal ..	Concurrent ..	23
18 ..	5 : 1 ..	Inverted ..	Concurrent ..	21

For Figures see next page.

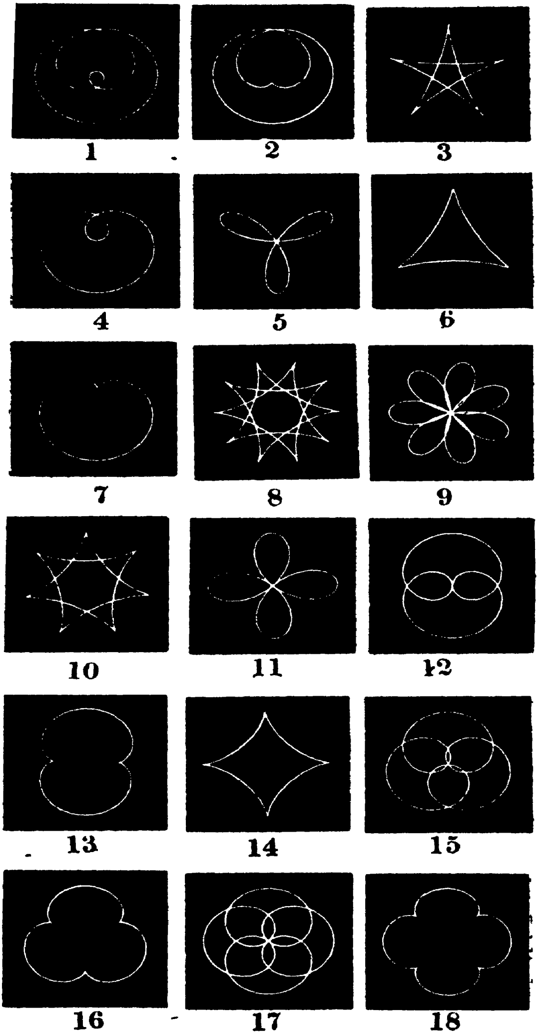


FIG. 52.

LAW OF NODAL PLACES.

A brief inspection of these figures will reveal a most important law, in accordance with which their general conformation is determined; it is this: The number of nodal points, loops or places in a counter-current figure is *the sum* of the ratio numbers; whilst in a concurrent figure it is *the difference* of the ratio numbers. These nodal places, it may be well to remark, are *all* that distinguish the compound figure from the plain ellipse. When the deflecting element is extremely weak, it is evident that the ellipse due to the main pendulum will not be distinctly modified—it will look like a plain ellipse, or circle, as the case may be. As the deflection increases the elliptic outline will become indented, showing alternate hollows and protuberances; stronger deflection will show acute hollows and sharp points; with yet stronger deflection the points will become loops; and when the main movement and deflecting movement are equally strong the deflections will meet at the centre of the figure. In counter-current figures these central points are the centres of the original hollows; in concurrent figures they they are the centres of the original protuberances.

Let me repeat that the alterations of outline here described are all due to alterations of *amplitude* only; the figure proper to any period-ratio may be so modified.

But in some cases it may quite easily happen that

the motion due to the deflector is *greater* than that which is due to the main pendulum; to understand the state of the case *then* we must consider for a moment its logical aspect.

Like the great majority of phenomena, deflectors and deflections are matters of comparison; the deflector is, or should be, *less* than that which is deflected. Therefore, when the motion due to the deflector becomes greater than that which is due to the main pendulum a logical revolution happens—the ratio is reversed; 3 : 1, for example, becomes 1 : 3; instead of the slower movement being deflected by the quicker, the quicker movement is now deflected by the slower. Although this is merely a question of logic, it is one of those questions which may easily lead to confused views, if not resolutely thought out.

I will now conclude my observations about the action of the T.E.P. with a few words in elucidation of some special features by means of which pendulum work may nearly always be distinguished from strictly mechanical work. These special features may be conveniently referred to as pendulum mutations. To understand what are the mutations to which pendulum figures are subject, we must be careful to remember what are the elements of these figures without such mutations; these are amplitude, phase and ratio. As long as the amplitude, phase and ratio of the constituent vibrations remain fixed the resultant figure will remain unaltered—the

vibrating body will continue to move through the same path. These three then—amplitude, phase and ratio—are the only elements that can be altered; but the alterations may be of such a character as to give rise to at least five distinct orders of mutation of the resultant figure. These five orders of mutation I will now endeavour to describe under the several titles of diminution, elimination, rotation, precession, and fluxion.

Pendulum Mutations.

Diminution is the natural decrease of amplitude produced by friction. Friction occurs chiefly at the points of suspension and at the pen-point, besides as atmospheric contact over the whole surface.

Diminution is, therefore, seldom equal in the various vibrations, and so gives rise to many changes of outline.

The diminution rate of a pendulum is a variable quantity, and, therefore, in most cases, unknown. Consequently the *ratio* of the diminution rates of the main pendulum and deflector is also unknown—except, of course, by experiment. The variability of the diminution rate depends on two other variables—strain and friction.

Strain varies with the pendulum mass, and with its structure.

Friction varies with the condition of the upper and lower regions of contact at the point of

suspension, and also with the mass of the pendulum, as well as with its vibration rate. Owing to extreme hardness the ball point is not likely to get worn ; but it may easily become rusty, causing an increase of friction ; or it may wear a rut in the steel bearing. This rut, if not exactly spherical, will resist the pendulum motion unequally in different directions, causing divers perversions of the main ellipse.

But, apart from the *condition* of the pendulum, the ratio of diminution rates varies considerably with the vibration ratio ; because the *work* of the deflector increases directly as it (the vibration ratio) increases.

In 4 : 1, for example, the deflector will do twice as much work in the same time (compared with the main pendulum) as it does in 2 : 1.

Practically, too, the diminution rate is modified by the *complexity* of the vibration ratio—apart from its magnitude—because a figure of simple ratio requires comparatively more pressure on the pen, to prevent the lines being too close. This increases, proportionally, the friction proper to the main ellipse.

This, however, is again dependent on the ratio of the *magnitudes* of the comparative masses.

There are yet other considerations affecting this question—such as the relation of the point of suspension of the deflector to the centres of gravity and of oscillation of the main pendulum.

The subject is not so simple as it may appear to be.

Elimination is a result of unequal diminution, whereby the various vibrations become successively exhausted; as an ultimate effect the figure may end in a straight line—the line of the surviving vibration. This is very apt to occur, as I have already explained,* when there is some decided infirmity at the point of suspension.

Rotation is an effect which becomes apparent when a figure, whose true ratio is complex, but nearly coincident with a simple ratio (29 : 10, for example, which is nearly coincident with 3 : 1), is worked out with a tolerably quick rate of diminution. In such a case the true figure (corresponding to the complex ratio) is never seen,† but the visible figure (that of the simple ratio) is repeated at regular intervals, either longer or shorter than its own precise period, and in slightly altered positions, thus seeming to rotate.

Precession is an effect produced in "rotation-" figures when the ellipses are elongated; it is caused by the alternate convergence and divergence of the major and minor axes. Some very beautiful effects of precession are often observable in 3 : 1 figures, especially in sharp concurrent specimens, but the flat varieties are also highly interesting. The curiously distinct character of development in

* See page 132.
† It would be distinctly seen if there were no diminution.

these 3 : 1 sharp and flat precession figures is, perhaps, the most remarkable feature in the whole range of twin-elliptic phenomena. The subject is too complex to be completely fathomed in this treatise, but I may here point out that in 3 : 1 concurrent precessionals "rotation" shortens the sharp, and lengthens the flat outline, and we find accordingly that the precessional groups of nodal points are shorter, as a rule, in sharp, and longer in flat specimens.*

Fluxion is the most convenient term I have been able to find to indicate an appreciable change of phase in vibration-figures arising from a definite inequality of ratio in the constituent vibrations of single ellipses. As this statement implies, fluxion may occur in either of the ellipses singly or in both together. As a definite and controllable phenomenon, fluxion may be induced in the deflector by simply elongating its mass. This is most conveniently done by constructing the deflector in the form of a pair of movable crossbars (see Fig. 53).

When the cross-bars are fixed at right angles the deflector swings at equal periods in all directions; but when there is any deviation from the right angles between the bars, the rate of vibration will

* PRECESSION IS ELLIPTIC OSCILLATION.—The expression may be strange, but the phenomenon is familiar to everyone who has watched the end of a vibrating rod. If the movement is quick enough the rod-end makes a continuous line—an ellipse. But the ellipse is not fixed; it wavers to and fro. So does the pendulum ellipse; so does the earth's orbit: that is precession. It is due to the fact that one of the vibrations constituting the ellipse is slightly quicker than the other. The difference is greater (and therefore the precession is more marked) in proportion as the ellipse is more elongated.

become slower in the direction of the acute angles and quicker in the direction of the oblique angles; consequently one vibration will gain upon the other and the ellipse will be changing phase continuously. The rate of change, or "fluxion," may be regulated to any nicety by the angle of deviation.

Any symmetrical form can be used for the fluxion deflector, if so shaped and divided that the mass can

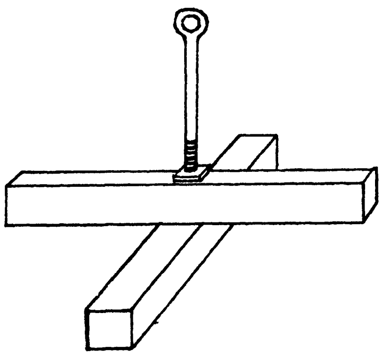


FIG. 53

be conveniently elongated. Fluxion in the main pendulum may be effected by extending the carrying board two or three feet on each side of the centre and placing the main weights on or near the ends.

Torsion. The foregoing observations are applicable to the T.E.P. when it is truly fixed and uniformly symmetrical.

In the great majority of cases, however, the distribution of the mass is *not* uniformly symmetrical, and the centre of gravity of the upper portion of the pendulum is not exactly in a straight line with the point of suspension of the deflector. Consequently there is more or less *torsion.*

When such torsion is very small its total effect may amount to a mere temporary deviation from the true course of the figure, which, after a brief period of cumulation, is followed by an equal reaction in the opposite direction. The torsion effect is then periodic, and may or may not synchronise with the effect due to precession.

Beyond a certain point, however (which will be different in every case), torsion will not admit of reaction in this periodic form. The very delicate balance of forces involved in this class of development becomes upset, and torsion is then continuous.

DIRECTIONS FOR USING GOOLD'S TWIN-ELLIPTIC PENDULUM.

THE first thing to consider is where and to what the Pendulum is to be hung.

The large hook with its ball-bearing point of suspension may be screwed very firmly into a wooden beam in the ceiling at a sufficient distance from the wall so as to allow of a good swing, or, if this is not convenient, a strong stout angle-iron bracket may be screwed or otherwise fixed to the wall.

It is very important that this suspension hook, by whatever method it is fastened, should be absolutely rigid, or the designs we hope to obtain will be spoilt by vibrations we do not require. Having secured the suspension hook, the next thing is to screw the large stirrup of the pendulum to the long rods ; if they are of metal, it will be found that one rod slides in a tube with a clamping screw, and at the other end of this is a smaller stirrup, inverted with a counter-sunk centre in it ; this centre will rest on the ball-bearing of the suspension hook. If, however, the rods are of wood, they will have to be bolted together and fastened with the nuts, the bolts passing through holes in the wood, thus allowing the pendulum to be lengthened or shortened as required.

Upon the large stirrup or table are placed the momentum weights, which are three in number and

made of cast iron. They must be so placed as to obtain an equal balance, as otherwise, when the pendulum is set in motion, it will not swing evenly, but will tend gradually to turn round. Where cost is of consequence the weights may be dispensed with, and ordinary bricks employed in their place.

On the top of the weights should be placed a stiff card or board, as this will give an even, flat surface to hold the cards on which the designs are to be made.

Underneath the table will be found a loop of twine; into this we hook one of the metal hooks from the other end of which we suspend the round heavy weight, with the fine adjusting screw, called the deflecting weight or deflector.

The next point to consider is the position of the penholder; this must be built up so as to bring the pen at right angles to the table. The penholder must have play on its two points, and must also be supported on something firm; a specially designed adjustable table, which will save the trouble of arranging any other support, can be supplied with the instrument if desired. The penholder should be placed in such a position that it is not likely to be shaken.

At one end of the penholder is a slot into which the small square weight with a pin in the centre of it must be placed; then comes the filling of the pen with ink; ordinary ink will not do, as it does not dry quickly enough, and as the lines cross and re-cross

one another a large blot would be the result; so a quick-drying ink must be used, which can be obtained in a variety of colours.

If a glass pen is to be used, choose a fairly coarse one, and care must be taken in filling it; it is also necessary to see that the ink has not been shaken up, or the pen will soon get clogged; do not put the ink in the pen, but put the tip of the pen in the ink, and suck it up with the mouth; by this method of filling the pens there will be no fear of getting lumps in which will not pass through the points. If, in spite of care, the pens should get stopped up, they can be easily cleared by placing them in nitric acid. The most obstinate cases will generally yield to nitric acid applied within and without, if assisted by a momentary touch of the gas flame. Care should be taken when pens have thus been cleared that before refilling them all traces of the acid be washed away, as it spoils the ink and may cause sore lips. Pens may often be cleared by merely touching the points with a damp card. When not in use they should be kept in clean water.

Having filled the pen, place it in the holder, keeping it in position by means of rubber rings, then place a few cards in the centre of the table, keeping them in position by a few small weights placed on the corners, and bring the pen forward so that the point comes about the centre of the card. It is necessary to attend carefully to the balancing

of the pen. If the pen presses too heavily on the card, the lines will be too wide apart, and the pen is apt to get stopped up with material from the surface of the card, which it will collect as it passes over the damp lines it has already made; if, on the other hand, the pen presses too lightly, the lines may run too close together, and will be broken where the pen has failed to make good contact. To adjust this the weight at the end of the penholder should be moved up the slot until something like an even balance is obtained; then place on the end which holds the pen one or more of the little weights to get a suitable pressure without too much friction on the card.

Having everything ready for action, card, pen, penholder and deflector all in place and all firm,

Take hold of the handle; you can easily move it so that any point on the paper shall come directly under the pen point—but it won't stay there. You can move the pendulum to any particular point, and beyond in a straight line, but you cannot move it in the form of any particular figure, unless that figure is proper to the ratio at which the pendulum happens to be set.

There is a great variety of figures proper to every possible setting of the pendulum; and amongst them, in every case, there are two principal types —con-current and counter-current. (See page 140.)

Now the most important figure that the pendulum can produce is the simplest, and the easiest to

obtain: aim at that first. Its ratio is 3 : 1—three elliptic movements of the deflector to one of the main pendulum.

To get the con-current form, try to swing the whole system in a circle.

Don't try too hard; but, after giving one or two circular impulses, watch the effect.

You don't know yet what the ratio is; but if the outline of the tracing covers a tolerably wide space on the paper, you can at once secure a beautiful figure—providing that you have not given too much swing to the deflector. But you are looking for the con-current 3 : 1 with two nodal points in the single outline: fix your attention on the *third* point (that will be the 1st in the repetition outline).

If the third point comes in front of the first, the deflector is too short; if after, it is too long.

By means of small hooks and fine adjustment screw you can, with a few trials, get the exact length; but, as a rule, the most beautiful figures are those which are acoustically "out of tune."*

To get the counter-current 3 : 1 first swing the whole system in a circle—as for the concurrent form—then, by means of a slight reverse impulse, break up the circular motion into a path with 4 external points. As before, watch the first repetition point (that will be the 5th in the

* With respect to this curious difference between acoustic and optic harmony, it is to be remembered that in twin-elliptic figures the eye has only *one interval* to consider, whereas in music the ear, to maintain its sense of tonality, must consider the relationships of at least seven intervals

continuous figure); if it comes *before* the 1st point the deflector will be too short; if after, too long.

It will be found that, under ordinary conditions, counter-current figures will develop more satisfactorily when the ratio is "sharp" (deflector too short), whereas concurrent figures are generally best when the ratio is "flat" (deflector too long).

This is a question of complexity, dependent on the fact that when the ellipses are elongated "precessional" effects (due to oscillation of the elliptic axes) are exaggerated by "rotation"; especially in the cases of concurrent sharp and counter-current flat figures.

Extra complexity demands extra care in balancing the conditions, but with proper conditions these very complex types produce the most beautiful effects.

Amplitudes. Every student who accepts the fact that the pendulum motion is constantly diminishing will occasionally be sorely puzzled by observing that at certain points the figure is undoubtedly getting larger. What is the interpretation of this paradox?

Close observation will show that the increase of dimensions in any one direction is preceded by an extra decrease in a direction at right angles to the increase. The total decrease is therefore constant, and, what is more, the decrease of the amplitude of each separate vibration is constant. What happens is that the elongated deflecting ellipse oscillates on

its axis—alters its direction, so O O O O and then back again ; so that its long and short axes alternately become parallel with the long axis of the main ellipse. The maximum enlargement of the figure is therefore coincident with the conjunction of the major elliptic axes.

But there can be no enlargement at any point of the figure unless the increase due to conjunction of major axes is greater than the decrease due to constant diminution of the separate vibration amplitudes. So that local enlargement of the figure only happens when the ellipses are much elongated or when the rate of diminution is slow.

To find the other ratios. Having fixed your pendulum lengths for 3 : 1, there will not be much difficulty in finding the lengths for other ratios. Suppose you want to get 8 : 3. Look at the ratio list (pages 135-139). In the order of magnitudes 8 : 3 is below 3 : 1 ; that is, 8 compared to 3 is less or slower than 3 compared to 1 ; therefore the deflector must be made longer.

Increase the length by adding a short hook. Now establish counter-current movement just as directed for 3 : 1. Drop the pen and watch. You are to look for a figure with 11 points in the course of 3 revolutions. Fix your attention on the 12th point, if it comes *before* the 11th (and within 3 revolutions), the deflector is too short ; if after, too long.

The outline is known to be complete when it begins to be repeated on a smaller scale.

To regulate ratios by weights.—It may happen that if you want to get 2 : 1 or some lower ratio, you will not have room to make the deflector long enough.

The ratio is determined not by absolute, but by comparative lengths of pendulums; and the lengths are measured from point of suspension to centre of oscillation.

You can decrease the virtual length of the main pendulum, and so increase the comparative length of the deflector, either by increasing the mass of the chief momentum weight or by decreasing the mass of the deflector. And vice versa.

Of course it may be more convenient merely to raise the chief momentum weight; but the whole range of the pendulum may be much extended by simply varying the masses of deflector and chief momentum weight.

Relative duration of deflecting vibration.—Some person may want to know how to ensure that the main pendulum and deflector shall come to rest at the same time. I don't know. Neither do I know that the problem is insoluble. Some clever mathematician may, perhaps, give us the solution. It seems to depend chiefly on the masses having a suitable ratio to each other; but much also depends on the condition of the points of suspension.

Principal Figures between the Octave 2 : 1 *and Double-Octave* 4 : 1.

(The comparative lengths of principal and secondary pendulums, measured from point of suspension to centre of oscillation, are inversely proportional to the squares of the vibrations.)

Ratio of vibrations.	Interval.	Comparative lengths.	
2 : 1	Octave	1	4
32 : 15	Minor 9th	225	1024
9 : 4	Major 9th	16	81
7 : 3	Harmonic Minor 10th	9	49
12 : 5	Minor 10th	25	144
5 : 2	Major 10th	4	25
8 : 3	Perfect 11th	9	64
11 : 4	Harmonic 11th	16	121
45 : 16	Augmented 11th	256	2025
3 : 1	Perfect 12th	1	9
16 : 5	Minor 13th	25	256
10 : 3	Major 13th	9	100
7 : 2	Harmonic 14th	4	49
32 : 9	Dominant 14th	81	1024
15 : 4	Major 14th	16	225
4 : 1	Double-octave	1	16

Plate XVI.

UPPER FIGURE.

3 : 1 COUNTER-CURRENT, AMPLITUDE-RATIO ABOUT 1 : 3 (INVERSE OF PERIOD RATIO).

The rates of diminution are varied by alternately adding small weights to, and removing them from, the penholder. The figure is almost exactly in tune– true to the ratio but the pendulum cannot refuse to encounter facts ; it must deal with them and register the result. It finds less friction in doing a short stroke than in doing a long one, and, therefore, it does the short stroke quicker That, however, does not alter the ratio, for both pendulums are under similar conditions. But the smaller pendulum does three strokes whilst the larger does one ; its velocity-gain is proportional and the ratio gets sharper. Had the friction on the pen been heavier the lines might have been much wider apart, the figure would have filled up quicker, and there would have been no perceptible alteration in the ratio.

The outline is Bazley's Fig. 26.

LOWER FIGURE.

3 : 1 COUNTER-CURRENT.

The ratio and initial phase are almost the same as above, but the deflection is stronger and, moreover, the deflector is heavier. Herein is illustrated the general character due to a heavy deflector. The initial outline is commonly more fanciful than usual, because further removed from a plain ellipse, there is also more "elimination" in the progress of the figure, producing apparent change of phase, but without actual "fluxion." (See pages 51 and 54). This point is worthy of careful attention, because change of figure from this cause may easily be incorrectly attributed to change of phase.

Plate XVII.

UPPER FIGURE.

3 I COUNTER-CURRENT WITH FLUENT DEFLECTION.

Again we have almost the same initial outline as in Plate XVI., but the change of phase is rapid, incessant and complete, beginning with counter-current and ending with con-current outline.

This is managed with the arrangement described on page 148.

LOWER FIGURE.

3 I SHARP CON-CURRENT, WITH PRECESSION.

In this figure we have a moderate example of the phenomenon of precession.

The alternate convergence and divergence of the major and minor axes of the two elongated ellipses acting in conjunction with a small amount of "rotation" (regulated by suitable adjustment of ratio) have given rise to two distinct series or systems of nodal points. It may be observed that the junction-patch between these two series is marked with a lacy envelope approximately circular in shape; this depicts the process of *turning the corner* by the nodal series, the two groups being set almost at right angles to each other.

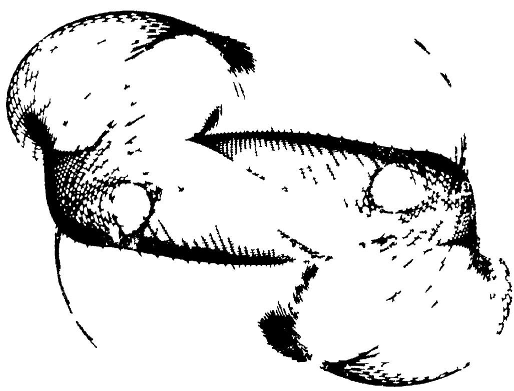


PLATE XVII

Plate XVIII.

UPPER FIGURE.

3 : 1 FLAT CON-CURRENT, WITH PRECESSION.

This, again, is a moderate example of precession. The initial outline is not far different from that of the preceding figure, but the filling up is much more coherent

There is no distinct junction-path between the nodal groups. Both this and the preceding figure have stopped short just where a new group of nodal points would have been initiated had the process continued, but, unless with a still finer pen, the lines would have run too close together But the accuracy of the pendulum movement can always be depended upon far beyond anything that the finest pen can register No mechanism not even Mr Austin's—can compete with the pendulum in this particular.

LOWER FIGURE.

3 1 SHARP COUNTER-CURRENT, RATHER A STRONG DEFLECTION.

There is no precession observable in this case, the two ellipses being almost circular, which means no distinction of major and minor axes.

"Rotation" is active here, but it is obscured in the general result With less pen-pressure the lines would have run too close together, and the beautiful intersections would have been blurred, with greater pressure the intersection figures would not have been sufficiently pronounced

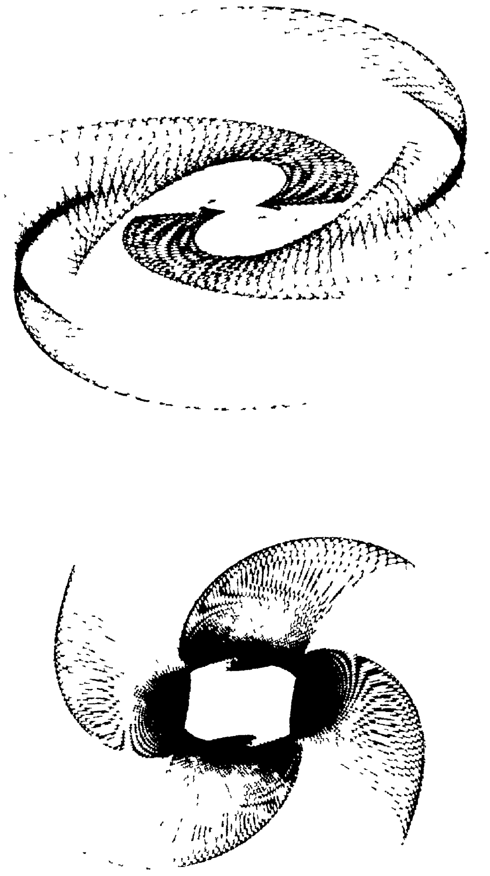


PLATE XVIII.

Plate XIX.

UPPER FIGURE.

3 : 1 FLAT CON-CURRENT.

This is a fair example of con-current "rotation." The ellipses are too circular to make any apparent precession.

LOWER FIGURE.

3 : 1 VERY SLIGHTLY FLAT CON-CURRENT.

The deflecting ellipse is elongated almost to a straight line. The chief feature in this figure is change of outline by elimination. The ratio here is so nearly exact 3 : 1 as to become ambiguous in its effects. The figure begins with a slightly flat deflection, and is just about to show a sharp turn where it leaves off.

Plate XX.

UPPER FIGURE

3 I SHARI COUNTER-CURRENT, FAINT DEFLECTION

The chief features of the figure are its four spiral envelopes marking the course of the four groups [illegible] se[illegible] [illegible]lal places 'Rotation' is here a decisive character but would have a more conspicuous effect [illegible] such with stronger deflection

LOWER FIGURE

I SHARP COUNTER-CURRENT, FAINT DEFLECTION SUPERPOSED

We have here two figures like the Upper Figure one over the other; both have the same amplitude, phase and ratio, but one has a slightly quicker rate of diminution than the other. A wonderfully beautiful effect is here produced with the simplest means conceivable.

Observe that the single figures here employed are precisely of the same class as the Upper Figure.

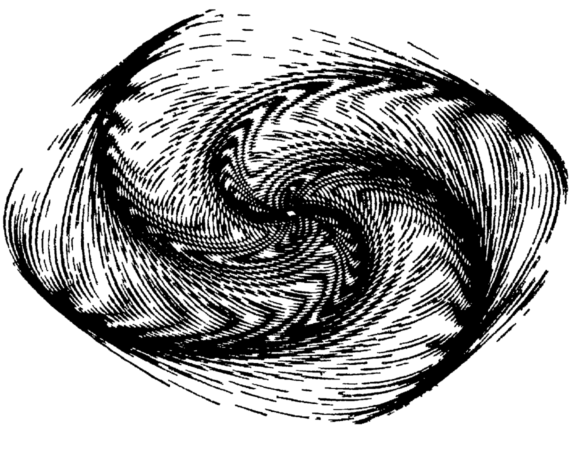


PLATE XX

Plate XXI.

UPPER FIGURE.

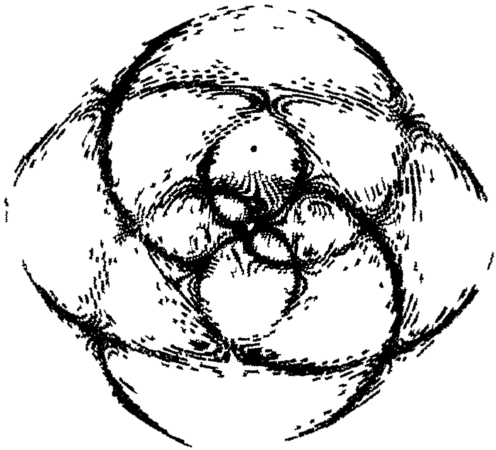
3 1 SHARP, FAINT, DEFLECTION, SUPERPOSED IN OPPOSITE DIRECTIONS.

Made like the last from two figures similar to (but not identical with) the Upper Figure in Plate XX, but the figures run in opposite directions Geometrically, this means exactly opposite phases.

LOWER FIGURE

3 1 SHARP, AC

Similar in all respects to the last, except that the deflection is slightly stronger, whilst the pen is finer and the pen point friction less, so that the lines are closer together. There is room for still more elaborate work in this direction, still finer pens, or diamond points on suitable films. The pendulum has hidden treasures in store beyond conception. All that is required is proper measures to make the pendulum movements apparent Look now at the Upper Figure on Plate XXII.



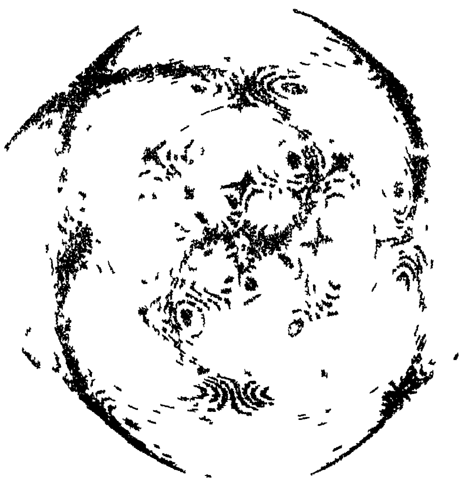


Plate XXII.

UPPER FIGURE.

3 : 1 SHARP, &c., AS THE PRECEDING.

The conditions that give rise to this figure are not far different from those that are operative in those on Plate XXI. Perhaps the pen is coarser and the friction greater, but the deflection is weaker and the ratio is more nearly exactly 3 : 1; in fact, one of the single figures is very slightly sharp, whilst the other is very slightly flat. Had *both* of the single figures been sharp, or both flat, then the eight intersection patterns instead of pointing straight to the centre would have had a spiral turn to right or left accordingly.

LOWER FIGURE.

47 : 17, COUNTER-CURRENT.

This is a clear case of "rotation" (see page 14). The figure with 64 external points is really not discernible, but if the outermost points be counted exactly in the order of their angular position, the precise number will be found in their proper places.

Had the pen-friction been much less, the amplitudes would have been more fully maintained, and the 64 points would have stood forward with almost equal prominence.

What the figure makes most readily apparent is the ratio of 11 : 4 sharp.

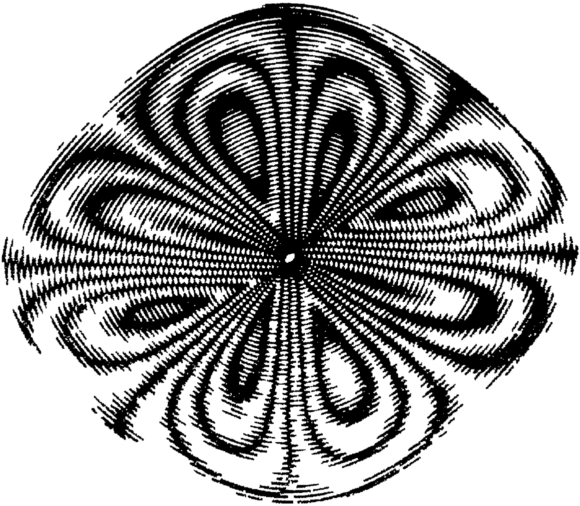


Plate XXIII.

UPPER FIGURE.

PERFECT FIFTH 3 : 2 COUNTER CURRENT

Still more difficult to manage than [illegible], it requires a still longer deflector. This corresponds to Bazley's No. 200 outline.

LOWER FIGURE.

MAJOR SIXTH 5 : 3 COUNTER CURRENT

One of the most interesting figures obtainable on account of its symmetrical outline, but difficult to produce and to control, because of the great length required for the deflector, about three times as much as in [illegible]

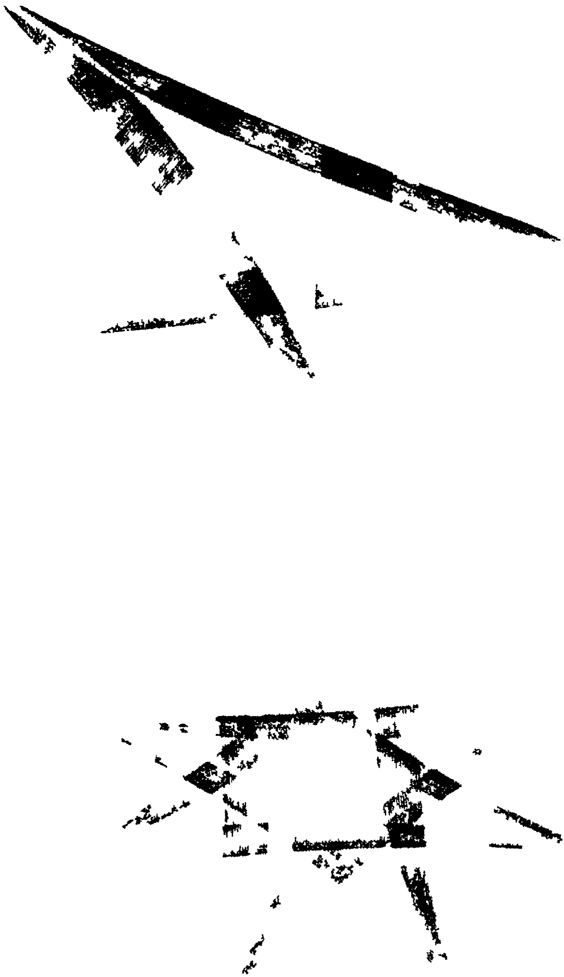


PLATE XXIII

Plate XXIV.

UPPER FIGURE.

7 : 3 COUNTER-CURRENT, EXACT RATIO.

The ratio is as nearly exact as the pendulum can make it, but it may be observed that the interaction patterns have a slight curvature, first to the right and, later, to the left. This means that the ratio is at first flat and becomes sharp towards the end. Some critics regard this sort of thing as a defect in the pendulum. It can only be a defect to those who require the pendulum to do the impossible. To those who would have the pendulum tell the truth it is an invaluable advantage.

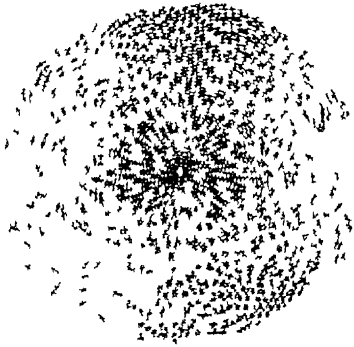
This ratio is one of the most prolific in good symmetrical figures, both con-current and counter-current.

LOWER FIGURE.

PERFECT ELEVENTH, 8 : 3 COUNTER-CURRENT, EXACT.

As before, only as exact as possible.

There is here a wonderful and beautiful display of the effects produced by minute variations of positive and negative forces, too small or too obscure to be amenable to scientific treatment. The interaction-patterns have at least a triple curvature, showing first a flat tendency, then sharp, then flat again. Beside this there is a distinct and spontaneous alteration of the amplitude ratio, showing comparatively stronger deflection towards the end, and giving rise to the eleven-rayed spiral series of nodal groups in the centre of the figure. Points such as these depend mostly upon the passive energies of the system, such as the condition of the point of suspension and the framework of the building supporting the ceiling-hook.



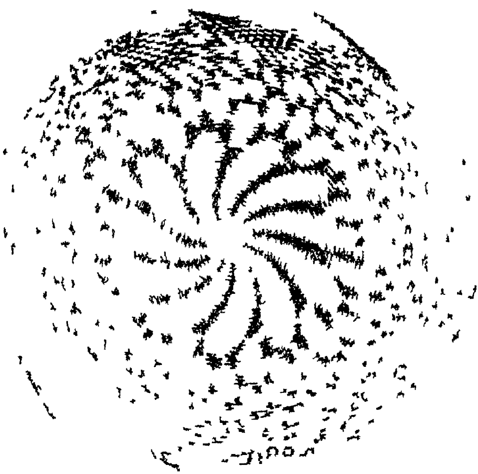


Plate XXV.

Austin's Figures.

These are unison figures, corresponding to the straight line and the circle (the two limits of the ellipse) when the amplitudes remain constant. Here, the amplitudes are gradually changing, the one from 0 to 1, the other from 1 to 0.

In the upper figure the two vibrations start together from the centre.

In the lower figure one vibration starts at the end of the path of the other.

Plate XXVI.

AUSTIN'S FIGURES.

The upper figure shows the "cusped octave" combination of vibrations, the parabola, with equal amplitudes. Here, the amplitudes are constantly changing from 0 to 1 and from 1 to 0. The result is a continuous series of parabolæ between the limits of two straight lines at right angles.

The lower figure shows the figure of the "perfect fifth" interval (ratio 3 : 2) under similar conditions to the above, beginning and ending with straight lines at right angles.

PART III.

VORTEX PLATES, SYNCHRONOUS PENDULUMS AND OTHER APPARATUS FOR VIRBATION EXPERIMENTS.

VORTEX PLATES.

By JOSEPH GOOLD.

NODE-FIGURES (commonly called "Chladni's sand-figures") are not vibration-figures, but vibration-boundaries or axes of vibration. Such are the lines made visible on vibrating plates, discs, membranes, etc., by loose particles of sand, or any tolerably heavy powder, scattered on the surface. These particles are thrown by the regions of strong vibration to the lines of least vibration. But if the particles are not heavy enough, they refuse to be thrown, just as a feather might refuse to be thrown; instead, they remain as little clouds in the air whilst the vibration is going on, and when the vibration ceases they descend in patches on the surface, thus distinguishing the positions where vibration is strongest. When the plates are regular the figures are regular, consisting chiefly of straight lines; but if the plates are un-even in shape and variable in thickness or in quality, the figures become correspondingly curved and complicated; and it is sometimes quite astonishing to find what strange variations in the figure are produced by very slight modifications in the symmetry of the plate.

It is important to observe that the prevalence of these modified figures becomes greater in proportion to the thinness of the plate; the reason being that irregularities in the thickness become greater in proportion as the thickness diminishes. It is for

this reason that node-figures on membranes, films, etc., such as paper-discs, drum-heads, and soap-films, are mostly void of straight lines ; but on fluid surfaces, such as water and mercury, they are easily made visible, the film or "skin" in these cases being absolutely regular.

The ten diagrams given in Fig. 54 represent some of the principal figures to be found on square plates. They are numbered in the order of pitch—dimensions being equal.

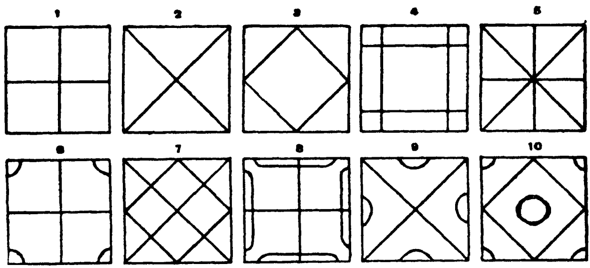


FIG. 54.
TEN OF THE PRINCIPAL NODE-FIGURES ON SQUARE PLATES.

Many other figures may be elicited from suitable plates, with pitches intermediate to those of the figures here given, and also with higher pitch.

But it is to be observed that the figures here shown are all uniformly symmetrical—symmetrical on all sides ; whereas it will be found that most, if not all, of the figures of intermediate pitch are only respectively symmetrical, and moreover these intermediate figures are generally very indefinite in

outline. The reason for this important fact is not far to seek.

Consider Fig. 55, for example. If the plate be symmetrical in form and homogeneous in texture, there will be no predisposing cause to determine which way of the plate the figure shall lie—whether with the single line from B to D, or from A to C.

Actually, the problem will be (in most cases) determined by accident; but in the struggle of the disposing causes the figure will, more than likely,

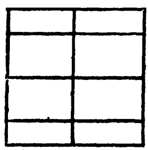


FIG. 55.

become distorted and come out as a curvilinear compromise.

It is for the reason just stated that figures of respective symmetry (symmetrical on two sides only) are generally more definite on plates of corresponding symmetry, and not on square or round plates.

On round plates the figures are either circular or radiating, or both.

It is said that the comparative rates of vibration for figures 1, 2, 6, and 9, shown above, are as 2, 3, 10, and 11 respectively. There may be some

mathematical basis for the statement, but I have not been able to verify it practically.

METHODS OF EXCITEMENT.

The usual method of excitement as described in acoustical treatises is by means of a 'cello-bow drawn vertically downward across the edge of the plate. This method is very effective and satisfactory in certain cases where the plate is *fixed by its centre* on a strong, convenient stand; but these conditions are in many cases non-admissible, particularly where the figure is open at the centre.

A method capable of much more extended application is that which I had the good fortune to discover some years ago.

In this method the necessary excitation of the plate is produced by gently rubbing it on any one of its ventral segments with a specially constructed vibrator which has been described as a "synchronising sound generator."

It consists essentially of an adjustable rubber which can be made to vibrate at any required rate—of course, within certain natural limits. This little instrument, in its simplest form, *may be* nothing more than an elastic rod of any kind. In its most complex form it consists of three parts—the rubber, the vibrator, and the handle.

With certain easily-arranged variations of structure the instrument can be made available for all possible conditions.

The "rubber" varies chiefly in hardness: for low notes india-rubber is a suitable material; for notes of moderate pitch, cork or leather; for very high notes, cane or hard wood.

The "vibrator," from one to six inches in length may be of cane, steel or glass.

The handle, of any convenient material, needs only that its mass should be greater or less in approximate relationship to the mass of the body to be excited.

In operation, the vibrator is first tuned as nearly as possible to the pitch of the note to be elicited. In other words, the vibrations of the rubber are made to synchronise with those of the plate or bar or whatever sonorous body is being investigated. Very much depends on this preparatory work, which is effected by lengthening or shortening the vibrator until sufficient accuracy of synchronisation has been obtained.

When this has been satisfactorily accomplished there will be no difficulty in eliciting the required note by rubbing a ventral* segment with the "generator" held at a slightly oblique angle to the plate in front of it.

One precaution must be attended to: the plate should be supported by means of two or more node-rests placed accurately beneath such node-lines as are found convenient, as in Fig. 56.

* A ventral segment is any portion of a vibrating body between two successive node-lines

The node-rest may be a wedge of cork or india-rubber. When an elastic plate, bar or tuning-fork is thus made to vibrate by synchronisation a much purer and stronger note can be elicited than that which would result from ordinary percussion.

The fact is that a synchronising rub is a continuous series of small percussions, so timed that each one follows its predecessor exactly at the moment when its energy will be most effective in strengthening the desired vibratory action. This is what is called " Cumulative action," the energy of

FIG. 56.

each small percussion being added to the sum of the preceding total. By this means the whole energy of the rub is guided into the synchronising vibration of the plate, and the tone has maximum power; whereas, when the same amount of energy is thrown into a single blow, it becomes distributed amongst many forms of vibration, and the resulting tone is mixed in quality and defective in duration.

In bars and rods the node-lines are all at right angles to the length of the bar, and the rates of its different forms of vibration are inversely as the squares of the successive odd numbers 3, 5, 7, 9, etc.

It is the same with long plates, except that when

the plate is wide enough it has another node-line through the middle, lengthwise. (See Plate I., Frontispiece, Figs. 1 and 2.)

A plate 34ins. by 4ins. by ½in., such as is commonly used for vortex demonstration, will have at least 12 sets of node-lines across its width, each set having its own fixed rate of vibration. Beside these there are other forms of vibration with one and sometimes two node-lines along the whole length of the plate. Altogether such a plate will yield from 20 to 30 excellent notes well within the ordinary range of hearing, and dividing itself across its node-lines in at least a hundred places.

For the sake of convenience we will distinguish the ordinary forms of plate-vibration, with lines across the width only, by the term "normal"; those with one additional line along the length we call "dual"; and those (to be described presently) in which the plate vibrates *transversely through its width* we shall refer to as "lateral" vibrations.

(Observe that we are not dealing at all with "longitudinal" vibrations.)

Considering only the first six normal forms of vibration and the first three dual forms, the node-lines on one half of a steel plate 34ins. by 4ins. by ½in. may be represented by the diagram on page 170.

It should be observed that all the even-numbered normal forms of vibration and all the odd-numbered dual forms have a node on the central line of the plate; and also, that number one normal node-line

is (practically) identical in position with the middle line of No. 5 normal.

It is not to be supposed that a plate will vibrate across all these node-lines simultaneously; indeed, it is commonly understood that plates, bars, and tuning-forks vibrate only across *one set* of node lines at a time; but neither of these views is precisely in accordance with the facts. It may be that the tone of a tuning-fork is pure enough for ordinary hearing purposes, just as good spring water is pure enough for ordinary drinking purposes; but there

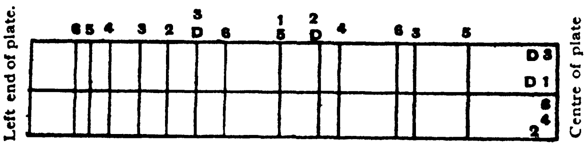


FIG. 57.

PRINCIPAL NODE-LINES ON ONE HALF OF A VORTEX PLATE 3¼ INS. BY 4 INS. BY ½ IN.

are purposes for which their impurities completely unfit both of them. In fact there can be no doubt that in many cases where we appear to be getting one form of vibration only, we are really hearing the joint effect of many.

It is also easily observable, in connection with these plates, that where an outside vibration is not *positively* modifying the form under special observation, it may yet exert a powerful influence by a kind of "passive resistance." In other words, the very possibility of one form of vibration may

sometimes make the development of another form impossible beyond a certain degree of intensity.

The vortex vibration in a well-balanced plate is brought into operation by rubbing the plate laterally—on its side edge—with a synchronising generator of suitable pitch. If a little chalk-powder be first sprinkled over the surface of the plate, it will be thrown by the vibratory motion into a form like that which is shown in Fig. 3, Plate I., Frontispiece.

The chalk particles on the vortex patches will be whirled round and round continuously as long as the vibration is maintained at a moderate intensity. The direction of rotation may be either clockwise or anti-clockwise, sometimes both; the vortices working in opposite directions.

CRAWLING CHAIN.

If, instead of chalk, a small chain be thrown on the vibrating plate, it will immediately settle itself on the curved line between the vortices and, with a continuous wriggle (varying in character with different plates), will crawl away to the nearest vortex and there coil itself up like a serpent, continuing to rotate as long as the plate remains sufficiently excited. Any small article placed on the vortex will be made to rotate in a similar manner.

At present the precise nature of vortex action in bars and plates remains an unexplained mystery.

There can, however, be no doubt that the phenomena depend chiefly upon the co-existence of certain special ratios between some of the most important forms of vibration of which any particular plate or bar may be capable. Principal amongst these important forms of vibration are: No. 1 lateral and No. 2 dual; and probably next in importance are 5, 4 and 6 normal (See Fig. 57).

It is not often that any of these are recognised as coming distinctly into simultaneous activity. But effects which are inaudible are not, therefore, invisible; and there can be very little doubt that several of these vibration forms do occur simultaneously with sufficient power to produce visible, if not audible, effects. But effects which are audible may yet be indistinguishable. *Something* is audible, for example, when the bottom notes of a common piano are struck; but there are not many persons who can distinguish the separate "harmonics" that take the place of the atrophied, if not quite absent, fundamental vibration.

And so the fact remains that it is only by carefully and laboriously adjusting the equilibrium of their constituent vibrations, by means of suitable clamping, cutting, drilling, gauging, grinding, etc., that these vortex plates can be made to act satisfactorily. Never yet in any single instance has a perfect plate been dropped upon by accident.

This does not mean that the phenomena are not perfectly natural; but only that there are enormous

chances against securing the requisite balance of dimensions even with the aid of experience, foresight and careful calculation.

Note.—*See also pages* 94 *to* 96.

VARIOUS EXPERIMENTS WITH VORTEX PLATES.

Besides the experiments of rotating vortices and the crawling chain already mentioned, strong vibrations, such as those which give rise to the lowest note of a 15lb. vortex plate, may be conveniently employed to illustrate many interesting transmutations of energy. One of the most readily available of this class of experiments is to be found in the use of

RESONANCE DISCS.

A piece of common note-paper simply held over the vibrating plate may be sufficient for a number of experiments; but in order to have the conditions more thoroughly at command it is necessary to have some ready means of altering the tension of the paper, or whatever form of membrane is employed.

For this purpose the paper, or other membrane, should be stretched on a frame of any kind. A circular frame is best, and if a large ring six or eight inches in diameter—just fitting inside the frame—is held in position on the paper by means of three tension screws the whole arrangement will be found very convenient.

Hold the stretched membrane over the vibrating plate—say, at six inches distant. If the tension of the membrane, or some considerable portion of it, be such that its vibration-period is the same as that of the plate, a low hum will be at once heard, and this will increase in power as the membrane is brought nearer; when almost close to the plate the resonant effect will become a loud blast varying in quality according to the condition of the membrane.

Should the proper tension not at first be found it can easily be arrived at by a few turns of the tension-screws. Some care will, of course, be required in order to secure evenness of tension throughout, but this is not always desirable. The effect of uneven tension in the resonant membrane is to increase the power of certain harmonics at the expense of the fundamental tone, thus producing indefinite variations of "klang," "timbre" or quality in the audible effect.

DUST-VIBRATIONS.

If a small quantity of very light powder, such as lycopodium, be thrown on to a vibrating resonance-disc, it will be shot into the air like a whirling nebula, and will then descend on the disc in the form of a multitude of vibrating mounds, all in a state of intense activity, like minute volcanoes, spurting forth streams of agitated particles. Further, if the vibration be equally maintained, these little active mounds will be seen

to gravitate towards each other, gradually coalescing and becoming larger, until at last the whole multitude have been absorbed in one central heap. Have we here any clue to the mystery of gravitation?

THE OSCILLATING LEAFLET.

If a long, narrow leaf or strip of paper be held vertically over a vibrating plate, in such a manner that the lower edge of the paper comes lightly in contact with the plate, the paper will oscillate to and fro as long as the vibration is maintained at the requisite intensity.

The paper is best held by its top edge in a clip fixed to a retort-stand. This simple phenomenon affords a real puzzle to the great majority of observers. This is what happens. Each vertical up-throw of the plate-vibration increases the slight bend of the paper at the line of contact; thus forcing some parts of its mass into a more forward position in the direction of the convexity of the bend. The following down-throw and up-throw of the plate acts so much quicker than the unbending of the relaxed paper that the latter is caught by the up-throw in a slightly advanced position; and then the same process is repeated until the accumulated energy of the paper bend enables it to act more quickly than the down-throw of the plate-vibration; the two operations are thus reversed, and the bend-energy discharges itself by carrying the paper-edge to the further side of the oscillation—and then the whole process is repeated.

VIBRATION-TOPS

are any light bodies (any shape) with one or more legs, all slightly inclined to the vertical in the same direction. They may be conveniently made of ordinary writing paper, say, an inch square or less, with strips cut nearly off on each side, and then bent down at right angles, so that the whole looks like a little table on four legs. When these are placed on a vibrating surface—such as a vortex-plate or a resonance disc, they will rotate in that direction to which the legs are inclined. If the "top" has only one or two legs, it will require the support of an axis fixed in the plate.

The action of the plate on the leg of the "top" is precisely the same as that which occurs with the "oscillating leaflet" described above, with this difference: the body of the leaflet is fixed to its support, whereas the body of the "top" is free. The forward motion of the foot is therefore continuously imparted to the body, and the effect of that is rotation, when the feet are ranged around a centre.

MELDE'S SPINDLE.

The term is used to describe the appearance presented by a vibrating string or cord. In Melde's well-known experiment the string is set in vibration by fixing one end to a prong of a large tuning-fork; it is stretched by passing over a fixed pulley-wheel, suitable weights

being fastened to the other end. A large vortex plate will be found even more convenient than a tuning-fork for this experiment. One end of the string should be fixed to the end of the plate by a small clamp or otherwise, the other end being held vertically over the plate (about 3ft. distant) by means of a long rod of bamboo or other light material. The string will divide itself into one, two, or more segments or "spindles" as the tension is varied.

In this experiment it is convenient to maintain the vibration of the plate by means of an electro-magnet, arranged to "make" and "break" its own contact with every vibration; but this is not essential, the phenomenon can be shown quite effectively by means of the ordinary "generator."

MERCURY VIBRATIONS.

are most easily shown by placing a shallow paper dish containing mercury on any portion of the plate where the vibration is not too strong. Care must be taken to have the surface of the mercury properly illuminated.

HARMONICS.

When resonance-discs are used in conjunction with the vortex plate, many harmonics of the fundamental tone are actively present in the audible effect, and can be identified by means of bars of nearly similar pitch, with which they will give audible beats.

INTERFLUXION FIGURES.

The chief beauty of Interfluxion Figures consists in their exquisite changes of form; they do not form a rigid figure as in Figs. 1 and 2 *on Plate I.*, but as one is watching them the *figures themselves go through rhythmic evolutions* which are graceful and fascinating in the extreme.

We have seen how vortex action is produced in steel plates of suitable dimensions by means of two systems of vibration working at right angles to each other. In that case the pitch-distance between the vibration systems needs to be small enough to admit of a distinct influencing of either one by the other, but not small enough to cause their actual synchronisation, nor even a near approach to that condition.

Interfluxion figures, on the other hand, are produced by the actual synchronisation of two vibration systems the best and most decisive results being obtained, in both cases, by systems No. 1 lateral and No. 2 dual. This is due to two principal facts: 1st, in consequence of the planes of vibration being at right angles to each other the resulting compound vibrations have a maximum freedom of motion; any other angle would involve greater loss of energy or less individuality of action.

2nd. Both of these systems have the centres of their principal ventral segments in the centre of the

plate. This also is largely conducive to freedom, as well as to symmetry, of the resulting compound vibration form.

In the case of vortex action (Plate I., 3rd figure from the top) it will be remembered that there is a fixed nodal line of varying curvature between the two vortex regions. With interfluxion figures (Plate I., 3 lower figures) there are no fixed lines; the fixed figures are only obtained by stopping the vibration completely.

As the two systems of vibration have the same "period" (= the same rate of vibration) it may be easily understood that neither of them can be excited without partially exciting the other; because every vibration of the one necessarily imparts a reduced impulse to the other. Consequently there is a continuous *interflux of energy* between the two systems, and any loose powder on the plate-surface is thrown into a wondrous labyrinth of ever-changing forms corresponding to the phases of the vibratory combination.

It is a most fascinating spectacle that these beautiful and sometimes startling evolutions of form present to the thoughtful onlooker. Some idea of the character of these natural wonders may, of course, be obtained from an inspection of the diagrams (Plate I., 3 last figures); but they can only be seen in their integrity whilst in actual motion, embodying and manifesting those marvellous interfluxions of energy which are involved in the

simultaneous development of two synchronising systems of plate-vibration.

So delicate and subtle is the balance of forces required for these demonstrations that no two plates, although identical in dimensions, will be found to act alike, and no method of manipulation which is found appropriate for one will prove equally suitable for another.

There is, however, a general resemblance in the phenomena exhibited by different plates, and in all cases the actual motions and configurations of loose particles scattered on the surface of the plate are surprisingly modified by minute alterations of its dimensions.

One particularly charming form of demonstration, which is generally attainable with a little perseverance, is that which I have designated as "The asteroid effect."

For this phenomenon it is generally necessary that the pitch-distance between the vibration-systems shall amount to about two or three beats per second; and if one of the systems exhibits too strong a tendency to throw the powder off the plate, that system must be checked or "damped" removing the node-rests to suitable distances from its proper node-regions.

All being ready, a strong excitement will cause a large portion of the loose-powder to move away from the central node-line and arrange itself in two nebular cloud-columns along the anti-nodal regions near the edges of the plate. Then wait.

As the vibration begins to die away the nebular columns shoot forth, from their inner edges, little whirling asteroids; few at first, they rapidly increase to a universal shower which ends by burying itself in the central node-line.

The "storm" can be maintained indefinitely by suitable manipulation.

THE GEOMETRIC PEN.

By RICHARD KERR.

Geometric Pen

PLATE XXVIII.

THE GEOMETRIC PEN.

BY RICHARD KERR.

MANY mechanical contrivances have been devised for producing geometrical figures by the compounding of two or more vibratory motions, some of them having been already mentioned in the earlier part of this work. There is one, however, which was designed by the late Mr. Pumphrey, and which in Plate XXVIII. Mr. Teasdale is shown in the act of working, which is included here for two reasons. Firstly, because it has not, we believe, been described in any other book; and secondly, because it is, perhaps, the most suitable for use by anyone who is fascinated by these designs and finds the arranging and drawing of them a delightful recreation.

Its advantages, from the point of view of one who uses it as a hobby, are that: It is impossible to exhaust its infinite variety, its powers of production being such that the leisure of a lifetime would hardly suffice to draw all the designs it is capable of producing.

It gives scope for artistic ability without requiring scientific knowledge or attainments, for while it is so simple that the modus operandi can be easily learned by anybody, plenty of room is left for the exercise of taste, care and patience.

It is interesting for onlookers to watch, as they can see the designs being formed from start to finish.

Any pattern desired can be reproduced at will if a note be taken of the position of each part of the instrument and of the wheels used. And finally it is portable, packing into a space less than 24 inches square and some 6 inches deep.

It can thus be readily taken to a friend's house or to a conversazione, where it always proves an attraction, although its scientific interest is so much less than attaches to the pendulum instruments.

The Geometric Pen consists essentially of a train of toothed wheels mounted on a solid base with extra wheels having various numbers of teeth, which can be substituted for others of the train when desired, and devices for otherwise varying the patterns, the possible variations amounting altogether to some millions.

A few of the designs drawn with this instrument are shown on page on a reduced scale, the original drawings being about 3 inches in diameter.

DESCRIPTION OF THE INSTRUMENT AND DIRECTIONS FOR USE.

Figure 58 shews a plan of the mechanical details of the original Geometric Pen now in the possession of the writer.

THE WHEELS.

The motion is obtained by the handle G on the bar F, which is attached to the axis of the wheels D and E. The teeth of all the wheels are alike, to admit of their being interchanged.

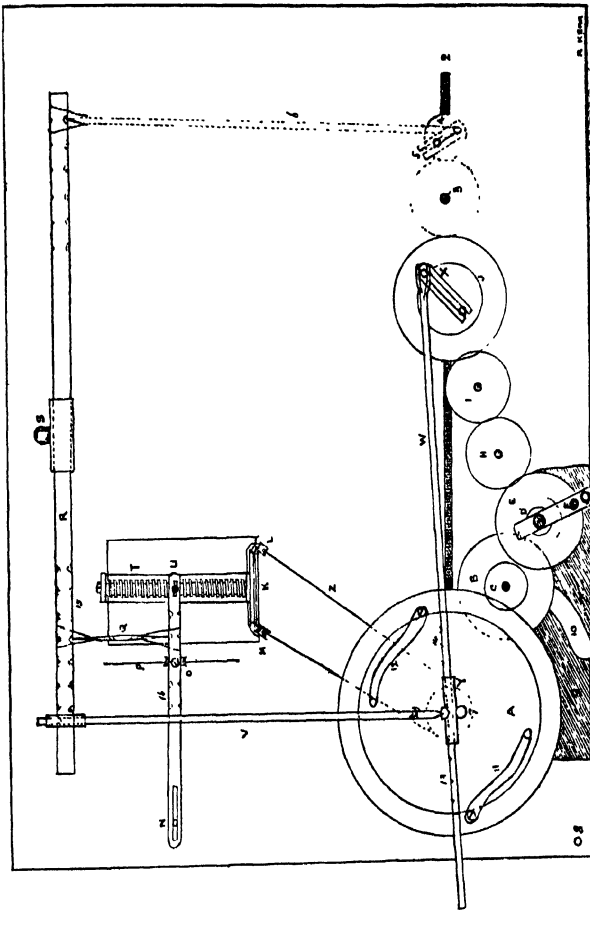


FIG. 58.

The upper wheel E has 36 teeth, the lower D has 20—hence, of course, the difference in their diameter.

These two wheels are never changed, the teeth of E engage those of H, while D engages B.

The wheel C, being attached to B, revolves with it and turns the table A, which carries the paper or card on which the design is to be made.

This wheel B can be easily removed and another with a different number of teeth substituted, so as to alter the speed of the table relatively to the other parts of the gearing. To accommodate the different-sized wheels thus required, a slot, 10, shewn in the plan, is provided in the wooden base to allow of the necessary adjustment.

The motion of wheel E is transmitted by wheels H and I to the crank-wheel J, which carries an adjustable crank, the throw of which can be regulated by means of a slotted bar. A rod W, about 16in. long, is fitted to the crank pin, and on this slides the penholder, 7, which can be clamped so as not to slip with the ordinary movements of the instrument.

If now the handle G be turned from left to right, or the direction taken by the hands of a clock, the rotating table A will take the same direction, while the crank-wheel J will move the opposite way.

THE TRAVERSE.

Attached to the underside of the rotating-table A is a wheel Y with grooved rim to take an end-

less cord or leather band Z, which goes under pulley L, over wheel K, and under pulley M. As the table A rotates it causes the band to rotate wheel K. This wheel K is fixed at its centre to the traverse screw T, which rotates as K is rotated.

The flat bar N U is free to move backwards and forwards at N. Underneath the end U is a half nut with a thread of the same gauge as that of the traverse screw T.

As the traverse rotates it gradually carries this half nut and its flat bar N U away towards T. In so doing it pushes the small rod Q, which fits at each end into small holes in the sides of the bars N U and R, and which is held in position by elastic bands, so that the bar R is pushed away gradually. Bar R is free to revolve on the pillar S, which can be clamped at any distance desired along R, the base being slotted for this purpose. The bar R carries the rod V, which fits into a socket on the other side of the penholder, 7.

In using the traverse the flat bar N U is liable to be "jumped." To prevent this it is gently held down by an elastic cord P which passes over a free pulley wheel O, and is attached at the baseboard at each end by loops.

It will now be seen that the pen is influenced by two motions, that of the crank to which the bar W is attached, and secondly, that caused by the rotating traverse screw. But in addition to these two motions the table A is rotating at the same

time. So that any figures produced by the pen must be the result of three motions.

VARIATIONS.

Now, without changing the first crank-wheel, let us see how alterations may be made and a variety of designs produced.

Suppose the wheel J is one of 48 teeth.

The sliding bar X may be moved along—say, the eighth of an inch.

Set the machine in motion, and a variation in the design is the result. Move X another eighth and a different design follows, and so on.

With alterations in this sliding bar alone, a number of variations may be made.

Again, the pen-holder may be moved along the bar W. Or the end of bar V may be taken out of the socket and put into any of the holes in the sides of the bar W, as at 13 and 14 in the plan. Each alteration again means a variation in the figure.

The bar V is attached to bar R by a clamp, and can be shortened towards the pen or lengthened. Each alteration again means a change in the figure.

By changing the position of the small bar Q and placing it in other holes in bar R as at 15, or in bar N U as at 16, quite a new set of figures may be produced, the rate of traverse being increased as Q approaches nearer to T. Now if any two or three of these alterations be made together, it

follows that the resultant figures must be quite different from any produced previously. So by varying these alterations a continuous change of pattern is obtained.

So far we have considered only one wheel, that with 48 teeth.

If we replace that wheel by another, say one of 36 teeth, quite a new set of designs will follow, and these will be enormously increased by making all the changes already named when the 48-toothed wheel was under consideration.

So by putting wheel after wheel at the first crank it is easy to see that the possibilities of the machine are very great indeed.

Up to the present we have only considered the traverse motions with one crank in operation.

PARALLEL OR 4-BAR MOTIONS.

Suppose we lift off N U and its half nut, and remove the small rod Q. This will render the traverse inoperative for the time.

We may now proceed to bring the parallel motion into play.

To the right of the first crank-wheel in the plan are two wheels, 3 and 4, a sliding bar 5, and another long bar 6. A narrow opening is cut in the top board running from 2 right to the rotating table A. The wheels J, 3 and 4, are fitted to pivots which come up through this slot, and are held firmly underneath by thumb-screws. These pivots can be

moved along to suit the sizes of the wheels used at the cranks.

Making sure that the bars N U and Q are removed, we now select two wheels, one for each crank, and they must be in ratio.

If we put 16 on the first and 48 on the second the figure produced will be in the ratio of 1 to 3.

Reverse these positions, and the figure will be as 3 is to 1. A great number of ratios may be introduced, and consequently a vast number of figures will be the result.

No alteration can be made in the machine without producing an alteration in the pattern.

There are holes made towards each end of the bar R, so that the ends of V and 6 may be fitted to any of these holes to increase or diminish the motion.

Elastic bands at the ends of these bars and at the penholder may take the place of universal joints.

Another point to be observed, which applies to both traverse and parallel motions, is the change that may be made by the wheel under C. The wheel marked B in the plan may have 40, 60, or 80 teeth. For a slow rotation of the table A a wheel with a large number of teeth will be necessary, and vice versa.

To the left of the table A is fitted a small brass folding bracket held in position by a brass hook, which will support the end of rod W when it is

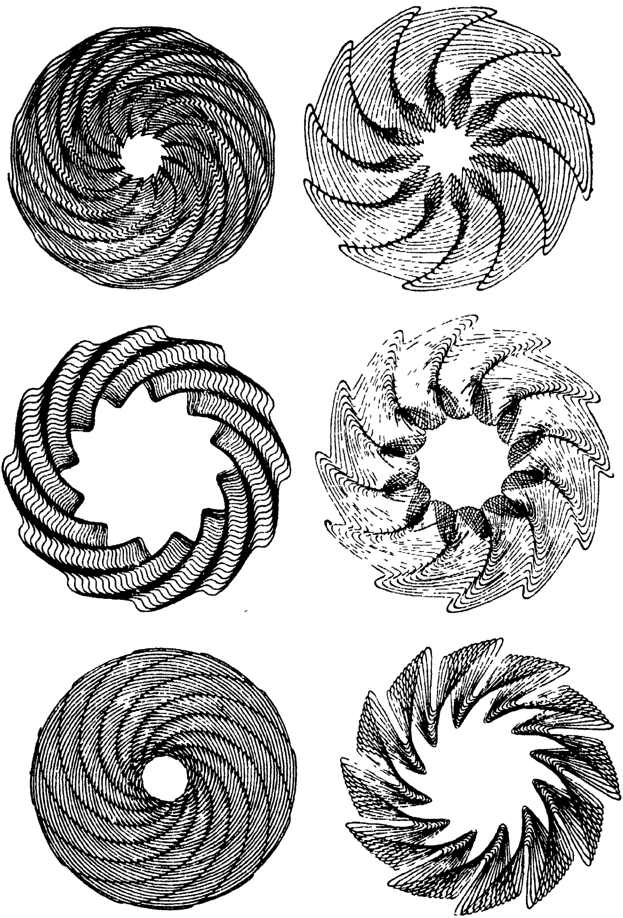


FIG. 59.

necessary to lift the pen out of the way in order to readjust the various parts of the instrument. This bracket takes the place of the upright rod shown in Plate XXVIII., and at 8 in Fig. 58, which was found to somewhat hamper the operator.

In Fig. 58 the second crank is shown in dotted lines, because with the traverse in its present position this crank cannot be used.

The machine must be used for traverse motion, or for parallel motion, but it is impossible to use both motions at the same time.

Of the two motions the traverse is the slower, but its designs seem to be more attractive.

In all cases any design produced should consist of one continuous line only.

The accompanying designs, Fig. 59, were all produced by the traverse motion with the 48-toothed wheel at the first crank, and the 60-toothed wheel at B.

The pen may be a glass tube drawn out to a fine point and carefully ground at the extreme end to allow ink, sufficient for a fine line, to flow out, as described in the chapters on Pendulums, or steel pens may be adapted to hold enough ink for a complete design. The pen should travel over the paper or card in a vertical position.

THE OPTICAL PROJECTION OF VIBRATION FIGURES ON A SCREEN.

THE OPTICAL PROJECTION OF VIBRATION FIGURES ON A SCREEN.

THE experiment of focussing a spot of light on a screen after reflection from mirrors fixed on a pair of large tuning forks is too well known to need mention here, but comparatively few know of the most fascinating and interesting method of showing this experiment devised by the late Mr. Lewis Wright, whose scientific attainments were highly valued by those who had the privilege of intimacy with him.

He employed a pair of reeds vibrating at right angles to each other and each carrying a small mirror, so that a beam of light forming a spot on the screen was reflected from one to the other in the same way as in the case of the tuning-forks. Either reed being vibrated, the spot was drawn out into a straight line on the screen, the one at right angles to the other, and if both were vibrated at once a moving living figure appeared on the screen, its form of course depending on the ratio between the two sets of vibration.

By mounting several reeds on a rotating drum so as to bring any one of them at will into position to work with another reed fixed at right angles to the drum, it became easy to obtain the characteristic figures of unison, octave and the intervening musical intervals.

By fixing the separate reed parallel to those on

the drum and allowing the light spot to fall on a rotating mirror before it finally reached the screen, "scrolls" can be opened out after the method employed by Tyndall, and "beats" can be projected.

This arrangement has several advantages over

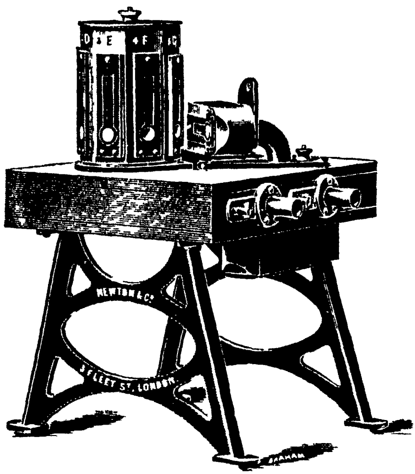


FIG. 60.

the tuning-fork method. In the first place a really good set of large tuning forks is an expensive luxury, and care is needed to keep them in good condition, failing which their value rapidly decreases; then the change from one ratio to another is far less troublesome with the reeds; and besides this the note given by each reed can be heard by the audience, which is

a great advantage. Many difficulties had to be encountered and overcome before this apparatus was really successful; special bellows had to be designed to obtain an equal blast, the reeds showed a tendency to "wobble" and spoil the figures; and the shape and size of the wind chest had to be determined by exhaustive experiment. Eventually by care, patience, and skill all these difficulties were overcome, and the result is a most perfect figure on the screen, which seems instinct with life, and which can be steadied or allowed to roll slowly or quickly through its phases at the will of the operator.

This apparatus, shewn in Fig. 60, was described by the inventor in his book "Optical Projection," but is included here also, as it is felt that it is hardly so well known as its merits deserve. We append the author's description of his method taken from the above work.

He says:—"Such an apparatus is far the best for the projection of compound figures, being superior to the most expensive forks in many respects. For its efficiency it is also far the cheapest. It possesses the following advantages, which are not found in combination in any other apparatus so far as I know.

"(*a*) It projects with ease *all* compound figures and scrolls. To project beats all that is necessary is to fix the separate reed perpendicularly the same as the other, and, having tuned, rotate the mirror to give the scroll.

"(*b*) All the notes are audible; *more so than* with forks.

"(*c*) Any note within the speaking range of the instrument can be added at any time for a few shillings.

"(*d*) Any interval can be changed for any other (so far as notes are provided) in half-a-dozen seconds. A complete set would comprise a lower C (for tenths and twelfths), two C's (for unison beats), and the diatonic scale C to c'.

"(*e*) There is absolutely no trouble in manipulation. Anyone blowing the bellows and having adjusted the pencil of rays once for all, the demonstrator has only to substitute notes as required, and manipulate a pinch-cock. If the pencil is awkward to manage, a black card with a circular aperture may be interposed between the first mirror and lantern to confine it within bounds. The mirrors are so near each other, that little light is lost, and the projection is brilliant.

"(*f*) The angular motion being great, the figure is on a large and bold scale

"(*g*) Most important of all, the notes *can be tuned in operation, with the greatest nicety.* There is no tiresome tuning or loading with wax. The pitch *varies with the wind-pressure,* within more then sufficient limits, and the hand on one or other of the pinch-cocks on

the tubes will either keep any phase of the figure "steady" or cause it to pass through the transitional forms with any rapidity desired.

With this apparatus Lissajous' figures become a delightfully easy and effective projection.

THE WILBERFORCE SPRING

AND

SYNCHRONOUS PENDULUMS.

THE WILBERFORCE SPRING.

BY PROF. L. R. WILBERFORCE.

IN this piece of apparatus an ordinary spiral spring is placed vertically, with its upper end rigidly fixed so that it can neither move up and down, nor turn round. A convenient way of satisfying this condition is to fix to a firm shelf a metal plate, on the opposite edges of which two little triangular notches have been filed, and to have the upper end of the spring bent into the form of a loop through which the plate passes so that the sides of the loop rest in the notches.

To the lower end of the spring there is rigidly attached a heavy cylindrical bob of metal furnished with four horizontal arms projecting symmetrically from it. These arms have screw threads cut on them, and on each arm a weight, acting as a nut on the screw, can be moved inwards or outwards. These weights are equal, and should be arranged so as to be equidistant from the axis of the bob.

If the bob is pulled vertically downwards for a short distance without twisting it and is then released, it of course starts by making up-and-down vibrations. It will be found, however, that these vibrations will not long maintain their original character, but that a twisting vibration superposed upon them will begin, grow gradually to a maximum, and then die away to nothing, after which the same changes will be continually repeated. By screwing the weights in and out the interval of time between

successive disappearances of the twisting vibration can be lengthened or shortened. If the weights are moved little by little they may be arranged so that this interval is twenty or thirty times as great as the time of one vibration of the bob, and it will then be noticed that the growth of the twisting vibration is accompanied by a marked decrease in the size of the up-and-down vibration. If the adjustment of the weights is carefully continued until the time between successive disappearances of the twist is as long as possible, it will be found that when the twisting vibration is at its greatest the up-and-down vibration will have for the time completely disappeared, and the successive growth and complete decay of up-and-down and twisting vibrations will then form a cycle of operations which it is very interesting to watch.

If on the other hand we start the bob by giving it a twist without any displacement upwards or downwards, it will behave in a manner similar to that which has been already described. That is, if the weights are out of the position of adjustment, the bob will at first execute pure twisting vibrations; then an up-and-down vibration will show itself, grow to a maximum, and die completely away, and this cycle will be continually repeated. It will be noticed that the growth of the up-and-down vibration will be accompanied by a diminution of the twisting vibration, which, however, will not in this case completely disappear, but only decrease to a cer-

tain minimum value. If, however, the weights are adjusted as described above, the twisting vibrations will die down to zero in this case also.

A general explanation of this seemingly anomalous behaviour of the vibrating body may be given by pointing out that when a spiral spring is lengthened or shortened, as in the up-and-down vibrations originally excited in it, small forces tending respectively to twist and untwist the spring are produced. The cumulative effect of these forces upon the suspended bob is to set up a gradually increasing vibration, and, in accordance with the doctrine of the conservation of energy, the amplitude of the up-and-down vibrations must decrease, whilst that of the twisting vibrations increases.

That the twisting vibrations of the bob are produced by the action of the small forces described above may readily be proved as follows.

Remove the upper end of the spring from its rigid fastening and suspend it from a short piece of plaited silk fishing line, so that this end of the spring is now free to twist. If the bob is pulled down and released it will make pure up-and-down vibrations which will now persist unchanged, while if the upper end of the spring is observed it will be seen to undergo a small twist backwards and forwards in time with these vibrations.

If a more complete explanation, covering all the facts observed, is desired, it cannot be obtained

without mathematical reasoning* which would be beyond the scope of this account, but the results of this reasoning may be summarised in the following manner.

A body attached to a spring in the way which has been described has two possible modes of vibration open to it, in each of which the vibration will persist unchanged if started, and in each of which the time of a complete vibration, or "period," to use the technical word, is definite.

Now it can be proved that, when the two periods differ considerably, one of the modes of vibration is a pure twisting motion of the body and the other is a pure up-and-down motion, and therefore if either of these two vibratory motions is given to the body it will persist unchanged.

If, however, the form of the suspended body is such that the two periods are not widely different, as is the case in the apparatus which we are now studying, the mode of vibration corresponding to each period is a twisting motion combined with an up-and-down motion, the resultant being a right-handed screw motion of a definite pitch for one period, and a left-handed screw motion, also of a definite pitch, for the other period.

That, if either of these vibratory screw-motions be imparted to the bob, the vibration will persist unchanged can be readily shown by experiment.

*Wilberforce, "Vibrations of a loaded spiral spring," Phil. Mag (5), Vol. 38, 1894, p. 386.

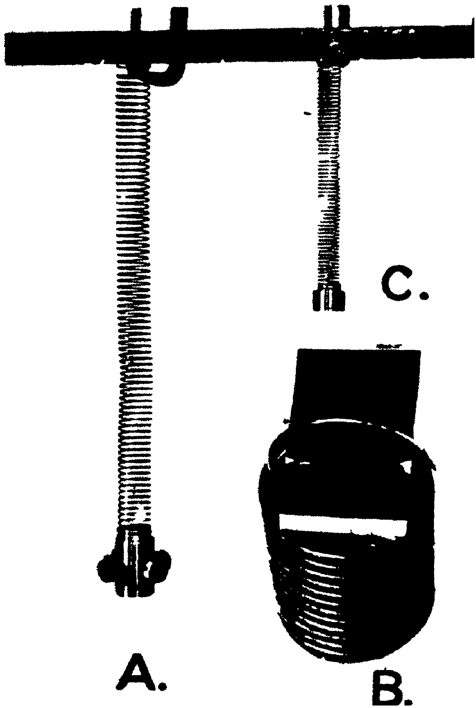


FIG. 61.

Each of these two motions can in turn be imparted by holding the spring lightly between the finger and thumb near the upper point of support and giving it very small right-handed or left-handed screwing motions in time with the vibrations which are at once set up. By trial the proper pitch of the screw motion to be given is soon found, and then a large vibration can be set up in which neither the up-and-down motion nor the twisting motion shows any alteration of amplitude.

If, however, the initial motion given to the bob is a purely up-and-down one, we are now in a position to understand why it does not persist unchanged. This form of motion is not natural to the system, as it is neither the right-handed nor the left-handed screw motion, which alone possess that character. It can, however, be considered as built up of two such vibratory screw-motions of suitable size existing simultaneously. These motions must, of course, have their rotations equal and opposite so as to neutralise each other, and so, as the one is right-handed and the other left-handed, the vertical displacements (which are unequal, if the pitches of the screws are unequal, as is in general the case) will be in the same direction. As the periods are different, one of these vibrations will gain on the other, the rotations will no longer balance, and a resulting twisting vibration will be noticed. When the one vibration has gained half a period on the other the rotations are in the same direction, and

the twist is a maximum, while the unequal displacements are in an opposite direction and the up-and-down motion is a minimum. When the one vibration has gained a complete period on the other, the original state of affairs is restored.

The behaviour of the bob when it is started with a pure twist vibration can be similarly explained. The two vibratory screw-motions now will have equal vertical displacements, which at first are opposite in direction, and as the one gains on the other, the effects already described will be produced.

Finally, it can be proved that when the two periods, which are always different, are made as nearly equal as possible the two screw-motions become of equal pitch, and consequently, if at any time the rotations neutralise each other, a time will come when the vertical displacements will neutralise each other, and *vice-versa.*

The complete transference of energy from up-and-down to twisting vibrations and back again in this case is very striking when observed. Another rather interesting way of exhibiting it is to have the cylindrical bob made without arms and movable weights, but adjusted to be exactly of such dimensions that the two periods shall be as nearly as possible equal. In this case the twisting motions of the bob are not readily seen by an observer at a little distance from it, and so, if an up-and-down motion is given to the bob, this motion gradually dies down to nothing, leaving the bob apparently at

rest, and then the motion recommences, grows, and decays again, in a way which seems very mysterious, until an explanation is furnished.

In the accompanying illustration, Fig. 61, A gives a general view of the spring and bob, B shows the way in which the upper end of the spring is held by the notched plate, and C represents a spring with adjusted cylindrical bob with which the experiment described in the last paragraph can be carried out.

SYNCHRONOUS SPRINGS AND PENDULUMS.

THE explanation of the transference of energy given by Professor Wilberforce in the preceding chapter applies equally to all so-called Synchronous Springs and Pendulums.

A simple but very striking form of synchronous

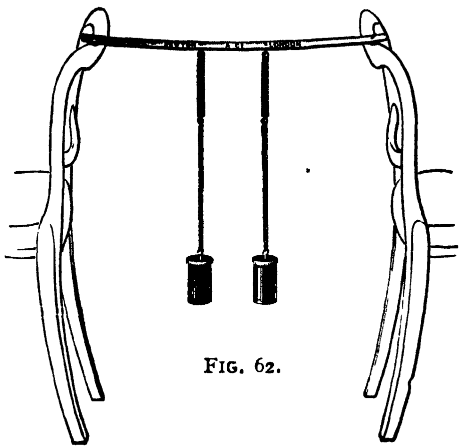


FIG. 62.

spring is shown in Fig. 62, and consists of an elastic wooden lath supported on the backs of two chairs, or in any more convenient manner. From this lath depend two spiral springs, to the lower end of each of which a heavy "bob" is attached.

If the weights of the "bobs" are correctly adjusted to the strength of the springs, and both of

these bear a suitable relation to the elasticity of the lath—a matter which requires some little care and skill to arrange—the transference of energy from one "bob" to the other is very striking. The apparatus is set in motion by pulling down one "bob" slightly so as to stretch the spring to which it is attached, and then letting it go suddenly. It naturally springs up and down when released, but the other "bob" also immediately starts vibrating in the same way, very slightly at first, but the amplitude of its vibrations rapidly increases, while that of the former diminishes till the second "bob" is moving with almost as much energy as the first had at starting, and the first is motionless.

No sooner has the energy been absorbed by the second "bob" in this way, than the process reverses itself and all the energy is again transferred to the original "bob." This is a most curious and fascinating experiment and always creates a great deal of interest.

A similar effect can be produced by two pendulums in a very simple manner. Two weights should be suspended by strings to two nails fixed securely a foot or so apart. If now the strings be looped round the ends of a wooden rod and one of the weights be set swinging by drawing it a few inches away from the other and then releasing it, the energy will gradually be absorbed by the second weight, and the first one will come to rest. The rate at which the transference takes place

depends on the position of the wooden rod, the nearer it is to the weights the quicker the change will be, and vice versa. It is possible to adjust the rod so near to the weights that the energy is entirely transferred after one vibration, so that each weight in turn swings once and then comes to rest.